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English Version

Eurocode 3 - Design of steel structures - Part 1-6: Strength and Stability of Shell Structures

Eurocode 3 - Calcul des structures en acier - Partie 1-6: Résistance et stabilité des structures en coque

Eurocode 3 - Bemessung und Konstruktion von Stahlbauten - Teil 1-6: Festigkeit und Stabilität von Schalen

This draft European Standard is submitted to CEN members for enquiry. It has been drawn up by the Technical Committee CEN/TC 250.

If this draft becomes a European Standard, CEN members are bound to comply with the CEN/CENELEC Internal Regulations which stipulate the conditions for giving this European Standard the status of a national standard without any alteration.

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EUROPEAN COMMITTEE FOR STANDARDIZATION
COMITÉ EUROPÉEN DE NORMALISATION
EUROPÄISCHES KOMITEE FÜR NORMUNG

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European foreword

This document (prEN 1993-1-6:2023) has been prepared by Technical Committee CEN/TC 250 “Structural Eurocodes”, the secretariat of which is held by BSI. CEN/TC 250 is responsible for all Structural Eurocodes and has been assigned responsibility for structural and geotechnical matters by CEN.

This document is currently submitted to the CEN Enquiry.

This document will supersede EN 1993-1-6:2007 and its amendments and corrigenda.

The first generation of EN Eurocodes was published between 2002 and 2007. This document forms part of the second generation of the Eurocodes, which have been prepared under Mandate M/515 issued to CEN by the European Commission and the European Free Trade Association.

The Eurocodes have been drafted to be used in conjunction with relevant execution, material, product and test standards, and to identify requirements for execution, materials, products and testing that are relied upon by the Eurocodes.

The Eurocodes recognize the responsibility of each Member State and have safeguarded their right to determine values related to regulatory safety matters at national level through the use of National Annexes.

0 Introduction

0.1 Introduction to the Eurocodes

The Structural Eurocodes comprise the following standards generally consisting of a number of Parts:

— EN 1990, Eurocode: Basis of structural and geotechnical design

— EN 1991, Eurocode 1: Actions on structures

— EN 1992, Eurocode 2: Design of concrete structures

— EN 1993, Eurocode 3: Design of steel structures

— EN 1994, Eurocode 4: Design of composite steel and concrete structures

— EN 1995, Eurocode 5: Design of timber structures

— EN 1996, Eurocode 6: Design of masonry structures

— EN 1997, Eurocode 7: Geotechnical design

— EN 1998, Eurocode 8: Design of structures for earthquake resistance

— EN 1999, Eurocode 9: Design of aluminium structures

— New parts are under development, e.g. Eurocode for design of structural glass

The Eurocodes are intended for use by designers, clients, manufacturers, constructors, relevant authorities (in exercising their duties in accordance with national or international regulations), educators, software developers, and committees drafting standards for related product, testing and execution standards.

NOTE Some aspects of design are most appropriately specified by relevant authorities or, where not specified, can be agreed on a project-specific basis between relevant parties such as designers and clients. The Eurocodes identify such aspects making explicit reference to relevant authorities and relevant parties.

0.2 Introduction to EN 1993 (all parts)

EN 1993 (all parts) applies to the design of buildings and civil engineering works in steel. It complies with the principles and requirements for the safety and serviceability of structures, the basis of their design and verification that are given in EN 1990 – Basis of structural design.

EN 1993 (all parts) is concerned only with requirements for resistance, serviceability, durability and fire resistance of steel structures. Other requirements, e.g. concerning thermal or sound insulation, are not covered.

EN 1993 is subdivided in various parts:

EN 1993-1, *Design of Steel Structures — Part 1: General rules and rules for buildings;*

EN 1993-2, *Design of Steel Structures — Part 2: Steel bridges;*

EN 1993-3, *Design of Steel Structures — Part 3: Towers, masts and chimneys;*

EN 1993-4, *Design of Steel Structures — Part 4: Silos and tanks;*

EN 1993-5, *Design of Steel Structures — Part 5: Piling;*

EN 1993-6, *Design of Steel Structures — Part 6: Crane supporting structures;*

EN 1993-7[1], *Design of steel structures — Part 7: Design of sandwich panels.*

EN 1993-1 in itself does not exist as a physical document, but comprises the following 14 separate parts, the basic part being EN 1993-1-1:

EN 1993-1-1, *Design of Steel Structures — Part 1-1: General rules and rules for buildings;*

EN 1993-1-2, *Design of Steel Structures — Part 1-2: Structural fire design;*

EN 1993-1-3, *Design of Steel Structures — Part 1-3: Cold-formed members and sheeting;*

NOTE Cold formed hollow sections supplied according to EN 10219 are covered in EN 1993-1-1.

EN 1993-1-4, *Design of Steel Structures — Part 1-4: Stainless steels;*

EN 1993-1-5, *Design of Steel Structures — Part 1-5: Plated structural elements;*

EN 1993-1-6, *Design of Steel Structures — Part 1-6: Strength and stability of shell structures;*

EN 1993-1-7, *Design of Steel Structures — Part 1-7: Plate assemblies with elements under transverse loads;*

EN 1993-1-8, *Design of Steel Structures — Part 1-8: Design of joints;*

EN 1993-1-9, *Design of Steel Structures — Part 1-9: Fatigue strength of steel structures;*

EN 1993-1-10, *Design of Steel Structures — Part 1-10: Selection of steel for fracture toughness and through-thickness properties;*

EN 1993-1-11, *Design of Steel Structures — Part 1-11: Design of structures with tension components made of steel;*

EN 1993-1-12, *Design of Steel Structures — Part 1-12: Additional rules for steel grades up to S960;*

EN 1993-1-13[2], *Design of Steel Structures — Part 1-13: Beams with large web openings;*

EN 1993-1-14[3], *Design of Steel Structures — Part 1-14: Design assisted by finite element analysis.*

All subsequent parts EN 1993-1-2 to EN 1993-1-14 treat general topics that are independent from the structural type such as structural fire design, cold-formed members and sheeting, stainless steels, plated structural elements, etc.

All subsequent parts numbered EN 1993-2 to EN 1993-7 treat topics relevant for a specific structural type such as steel bridges, towers, masts and chimneys, silos and tanks, piling, crane supporting structures, etc. EN 1993-2 to EN 1993-7 refer to the generic rules in EN 1993-1 and supplement, modify or supersede them.

0.3 Introduction to prEN 1993-1-6

prEN 1993-1-6 gives design requirements for steel shell structures that are subject to forces and pressures that induce membrane and bending stress resultants in the shell. It also covers annular plates and ring stiffeners. Its provisions can be used for a wide variety of stiffened and unstiffened curved structures through the application of computational methods. It is applicable to silos, tanks, chimneys, wind turbine towers, biodigesters and piles.

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[1] Under preparation.
[2] Under preparation.
[3] Under preparation.

0.4 Verbal forms used in the Eurocodes

The verb “shall" expresses a requirement strictly to be followed and from which no deviation is permitted in order to comply with the Eurocodes.

The verb “should” expresses a highly recommended choice or course of action. Subject to national regulation and/or any relevant contractual provisions, alternative approaches could be used/adopted where technically justified.

The verb “may" expresses a course of action permissible within the limits of the Eurocodes.

The verb “can" expresses possibility and capability; it is used for statements of fact and clarification of concepts.

0.5 National Annex for prEN 1993-1-6

National choice is allowed in this standard where explicitly stated within notes. National choice includes the selection of values for Nationally Determined Parameters (NDPs).

The national standard implementing prEN 1993-1-6 can have a National Annex containing all national choices to be used for the design of buildings and civil engineering works to be constructed in the relevant country.

When no national choice is given, the default choice given in this standard is to be used.

When no national choice is made and no default is given in this standard, the choice can be specified by a relevant authority or, where not specified, agreed for a specific project by appropriate parties.

National choice is allowed in prEN 1993-1-6 through notes to the following:

4.4 (3)	6.3.2 (3)	6.3.4 (2)	9.8.2 (12)

National choice is allowed in prEN 1993-1-6 on the application of the following informative annexes:

Annex A	Annex B	Annex C

1 Scope

1.1 Scope of prEN 1993-1-6

(1) prEN 1993-1-6 provides rules for the structural design of plated steel structures that have the form of a shell of revolution (axisymmetric shell).

(2) This document is applicable to unstiffened fabricated axisymmetric shells formed from isotropic rolled plates using both algebraic and computational procedures, and to stiffened axisymmetric shells with different wall constructions using computational procedures. It also applies to associated circular or annular plates and to beam section rings and stringer stiffeners where they form part of the complete shell structure. The general computational procedures are applicable to all shell forms.

(3) This document does not apply to manufactured shells or to shell panels or to elliptical shell forms, except that its computational procedures are applicable to all shell structures. This document does not apply to structures under seismic or other dynamic loading. It does not cover the aspects of leakage of stored liquids or solids.

(4) Cylindrical and conical panels are not explicitly covered by this document. However, the provisions of subclause 9.8 can be used provided that appropriate boundary conditions are taken into account.

(5) This document defines the characteristic and design values of the resistance of the structure.

(6) This document is concerned with the requirements for design against the ultimate limit states of:

— plastic failure;

— cyclic plasticity;

— buckling;

— fatigue.

(7) Overall equilibrium of the structure (sliding, uplifting, overturning) is not included in this document. Special considerations for specific applications are included in the relevant application parts of EN 1993.

(8) Detailed formulae for the simple calculation of unstiffened cylinders, cones and spherical domes are given in the Annexes.

(9) Provisions for simple calculations on specific stiffened shell types are given in EN 1993-4-1.

(10) This document is intended for application to steel shell structures. Where no standard exists for shell structures made of other metals, including high strength steels, the provisions of this document are applicable provided the appropriate material properties of the metal are taken into account.

(11) The provisions of this document are intended to be applied within the temperature ranges defined in the relevant EN 1993 application parts.

(12) Where no application part defines a different range, this document applies to structures within the following limits:

— design metal temperatures lie within the range −50 °C to +100 °C, except when using the special provisions given in 5.1;

— radius to thickness ratios (r/t) within the range 50 to 2 000;

— manufactured circular hollow sections according to EN 10210 and EN 10219 are outside the scope of this standard and are covered by EN 1993-1-1. However, if no other provisions are available, the rules of this document are useful for manufactured circular hollow sections. In particular, this document is applicable to the design of manufactured piles (see EN 1993-5) provided the imperfections and tolerance requirements of EN 1993-5 are adopted in place of those specified in prEN 1993-1-6, and where no other standard covers the specific pile geometry.

NOTE 1 Experimental and theoretical data relating to manufactured circular hollow sections were not considered when this document was drafted. The application of this document to such structures therefore remains the responsibility of the user.

NOTE 2 The stress design rules of this document can be rather conservative if applied to some geometries and loading conditions for relatively thick-walled shells.

NOTE 3 Thinner shells than r/t = 2 000 can be treated using these provisions but the provisions have not been verified for such thin shells.

NOTE 4 The maximum temperature is restricted so that the influence of creep can be ignored where high temperature creep effects are not covered by the relevant application part.

NOTE 5 Where temperatures outside the above range are involved, the thermally adjusted properties can be found in EN 1993-1-2 or other CEN standards as appropriate. Where no other standard is available, refer to EN 1993-1-2 which, though intended for the design of steel structures against fire, gives general temperature-dependent material properties that can be more widely used (see 5.1(10)).

1.2 Assumptions

(1) Unless specifically stated, the provisions of EN 1990, EN 1991 (all parts) and the other relevant parts of EN 1993-1 (all parts) apply.

(2) The design methods given in prEN 1993-1-6 are applicable if:

— the execution quality is as specified in EN 1090-2, and

— the construction materials and products used are as specified in the relevant parts of EN 1993 (all parts), or in the relevant material and product specifications.

NOTE The buckling-related tolerance requirements of this document differ in some aspects from those of EN 1090-2 (see Clause 9).

(3) The provisions in this document apply to materials that satisfy the brittle fracture provisions given in EN 1993-1-4, EN 1993-1-10 and EN 1993-1-12.

(4) In this document, it is assumed that wind loading, seismic actions and bulk solids flow can, in general, be treated as quasi-static actions.

(5) Dynamic effects are outside the scope of prEN 1993-1-6, and are covered by the relevant application part of EN 1993 or EN 1998, including the consequences for fatigue. However, the stress resultants arising from dynamic behaviour are treated in this part as quasi-static.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

NOTE See the Bibliography for a list of other documents cited that are not normative references, including those referenced as recommendations (i.e. in ‘should’ clauses), permissions (‘may’ clauses), possibilities ('can' clauses), and in notes.

EN 1090-2, *Execution of steel structures and aluminium structures — Part 2: Technical requirements for steel structures*

EN 1990, *Eurocode: Basis of structural and geotechnical design*

EN 1991 (all parts), *Eurocode 1: Actions on structures*

EN 1993 (all parts), *Eurocode 3: Design of steel structures*

ISO 8930, *General principles on reliability for structures — Vocabulary*

3 Terms, definitions and symbols

For the purposes of this document, the terms and definitions given in EN 1990, EN 1993-1-1, ISO 8930 and the following apply.

3.1 Definitions

3.1.1 Structural forms and geometry

3.1.1.1
base ring
structural member that passes around the circumference of the shell of revolution at the base and provides a means of attachment of the shell to a foundation or other structural member, needed to ensure that the assumed boundary conditions are achieved in practice

3.1.1.2
circumferential joint
joint that passes around the circumference of an axisymmetric shell

3.1.1.3
complete shell or shell assembly
shell composed of a number of shell segments (cylindrical, conical, spherical, etc.)

Note 1 to entry: In this standard, each segment of the shell assembly is assumed to be a shell of revolution.

3.1.1.4
constructional detail
part of a shell with a geometry that causes locally raised stresses relevant to the fatigue limit state (LS4), such as welded joints, bolted joints and connections.

Note 1 to entry: The geometric feature that causes the stress raising effect is also referred to as a “notch” in EN 1993-1-9.

3.1.1.5
course
set of rolled plates connected by vertical joints that make up a single layer of shell between horizontal joints

Note 1 to entry: Several courses of the same thickness can together become a strake.

3.1.1.6
fabricated shell
shell structure that is constructed by rolling plates into curved cylindrical panel sections and then assembling them by welding or bolting into a complete shell form

3.1.1.7
joint
line between two pieces of shell that are part of the same shell segment but fabricated from different pieces of shell plate

Note 1 to entry: A joint can be welded or bolted or connected in any other manner. The term "joint" is extensively used in shell structures, but it is used with a slightly different meaning from that found in EN 1993-1-8.

3.1.1.8
junction
line at which two or more shell segments meet

Note 1 to entry: A junction can include a stiffener, which can be treated as a junction at the circumferential line of attachment of a ring stiffener to the shell.

3.1.1.9
lap joint
joint in which the two shell plates overlap across the joint, increasing the total shell thickness locally

3.1.1.10
manufactured shell
shell or tubular member that is made in a factory by controlled processes in which the complete circular or elliptical form is achieved through folding, rolling or similar processes and using longitudinal or spiral welding

Note 1 to entry: Manufactured shells or tubular members are typically manufactured to meet the specifications of EN 10210 or EN 10219. Manufactured shells are outside the scope of this document except where permitted by 1.1 (3) and 1.1 (12).

3.1.1.11
meridian and meridional direction
line on a shell surface that lies in the plane through the axisymmetric shell axis

Note 1 to entry: The meridional direction is the tangent to the meridian at any point. In a cylinder, the meridian is parallel to the axis and the meridional direction is synonymous with the axial direction. In conical shells the meridian is straight but inclined to the axis. In other shell forms the meridional direction changes with axial position.

3.1.1.12
meridional joint
joint that lies on the meridian of an axisymmetric shell

3.1.1.13
middle surface
surface that lies midway between the inside and outside surfaces of the shell at every point, which is the reference surface for analysis, and can be discontinuous at changes of thickness or at shell junctions, leading to eccentricities that can be important to the shell structural behaviour

Note 1 to entry: In a shell stiffened on either one or both surfaces, the reference middle surface is still taken as the middle surface of the curved shell plate.

3.1.1.14
notch
position in a constructional detail where locally raised stresses arise that are relevant to the fatigue limit state (LS4)

Note 1 to entry: The term “notch” is widely used in EN 1993-1-9.

3.1.1.15
rib
local member that provides a primary load carrying path for bending down the meridian of the shell, representing a generator of the shell of revolution, used to transfer or distribute transverse loads by bending

3.1.1.16
ring beam or ring girder
circumferential stiffener that has bending stiffness and strength both in the plane of the shell circular section and normal to that plane, acting as a primary load carrying structural member and provided for the distribution of local loads into the shell

3.1.1.17
ring stiffener
local stiffening member that passes around the circumference of the shell of revolution at a given point on the meridian, normally assumed to have no stiffness for deformations out of its own plane (meridional displacements of the shell) but to be stiff for deformations in the plane of the ring, and provided to increase the stability or to introduce local loads acting in the plane of the ring

3.1.1.18
shell
structure or a structural component formed from a curved thin plate

Note 1 to entry: The curvature plays a vital role in its structural resistance and can be either in one direction (cylinder or cone) or two directions (spherical, ellipsoidal, toroidal, hyperboloid etc.).

3.1.1.19
shell of revolution
shell whose geometric form is defined by a middle surface that is formed by rotating a meridional generator line around a single axis through 2π radians

3.1.1.20
shell panel
incomplete shell of revolution

Note 1 to entry: The shell of revolution is termed incomplete if it has meridional boundaries that lie at circumferential locations less than 2π radians apart.

Note 2 to entry: Shell panels are outside the scope of this document except where permitted by 1.1 (4).

3.1.1.21
shell segment
shell of revolution in the form of a defined shell geometry, usually with a constant wall thickness but sometimes consisting of multiple strakes

Note 1 to entry: A shell segment can be a cylinder, conical frustum, spherical frustum, annular plate, toroidal knuckle or any other form of shell of revolution.

3.1.1.22
stepped wall
shell with a fixed geometric shape (cylinder, cone, etc.) in which different parts have different thicknesses to accommodate the variation of local resistance requirements

3.1.1.23
strake
zone of constant thickness within a shell constructed with a stepped wall

3.1.1.24
stringer stiffener
local stiffening member that follows the meridian of the shell, representing a generator of the shell of revolution, provided to increase the stability, or to assist with the introduction of local loads, but not intended to provide a primary resistance to bending effects caused by transverse loads

3.1.2 Limit states

3.1.2.1
buckling (LS3)
ultimate limit state where the shell structure suddenly loses its stability under membrane compression and/or shear, leading either to large displacements or to the shell being unable to support the applied loads

3.1.2.2
cyclic plasticity (LS2)
ultimate limit state where repeated yielding is caused by cycles of loading and unloading, leading to a low cycle fatigue failure where the local energy absorption capacity of the material is exhausted

3.1.2.3
fatigue (LS4)
ultimate limit state where more than N_f cycles of loading cause cracks to develop in any part of the structure, so that further load cycles can lead to rupture

Note 1 to entry: This limit state is termed "high cycle fatigue" in EN 1990. The value of N_f is defined in 6.3.4(2). EN 1993-1-9 has no provisions for numbers of cycles less than 10 000. For lower numbers of cycles involving high stresses, Clause 8 (LS2) is relevant.

3.1.2.4
plastic failure limit state (LS1)
ultimate limit state where the shell develops zones of yielding with combined membrane and bending deformations in a pattern such that its ability to resist increased loading of the same form is deemed to be exhausted

Note 1 to entry: Many ductile shell structures can continue to resist increased loading with extensive yielding and substantial changes of geometry. For these conditions, a limitation on deformation is used to define the plastic failure limit state.

3.1.2.5
tensile rupture (LS1)
ultimate limit state where the shell plate experiences gross section failure due to membrane tension

3.1.3 Actions

3.1.3.1
axial load
externally applied loading acting in the axial direction in an axisymmetric shell

3.1.3.2
axial compression
axial load inducing compressive membrane stresses in a cylindrical shell

3.1.3.3
external pressure
component of the surface loading acting normal to the shell in the inward direction q

Note 1 to entry: The magnitude of the external pressure can vary in both the meridional and circumferential directions (e.g. under snow, see EN 1991-1-3, or wind, see EN 1991-1-4).

3.1.3.4
global bending
actions causing a cylindrical or conical shell to bend as a complete structure about an axis normal to the axis of the shell

Note 1 to entry: This corresponds to a cosine variation (harmonic 1) of the axial stresses around the circumference of the shell and is equivalent to beam bending.

3.1.3.5
hydrostatic pressure
pressure varying linearly with the axial coordinate in an axisymmetric shell, which is deemed to have its axis vertical

3.1.3.6
internal pressure
component of the surface loading acting normal to the shell in the outward direction p

Note 1 to entry: The magnitude of the internal pressure can vary in both the meridional and circumferential directions (e.g. under solids loading in a silo, see EN 1991-4, or under sloshing pressures in a tank, see EN 1998-4, or tilt settlements under large diameter tanks).

3.1.3.7
local load
point applied force or distributed load acting on a limited part of the circumference of the shell and over a limited height

3.1.3.8
partial vacuum
uniform net external pressure, typically caused by the removal of stored liquids, solids or gas from within a container that is inadequately vented (see EN 1991-4)

3.1.3.9
patch load
local distributed load acting normal to the shell

3.1.3.10
radial load
externally applied loading acting normal to the surface of a cylindrical shell or normal to the axis in an axisymmetric shell

3.1.3.11
suction
uniform net external pressure q due to the reduced internal pressure in a shell (e.g. due to openings or vents under wind action, see EN 1991-1-4 or due to partial vacuum)

Note 1 to entry: The external pressure is given a separate notation q to simplify the provisions of this document. Otherwise many formulae would involve negative values of the outward pressure p.

3.1.3.12
thermal action
temperature variation either down the shell meridian, or around the shell circumference or through the shell thickness, or combinations of these spatial variations

3.1.3.13
wall friction load
meridional component of the surface loading acting on the shell wall due to friction connected with internal pressure (e.g. when solids are contained within the shell, see EN 1991-4)

3.1.4 Stress resultants and stresses in a shell

3.1.4.1
bending stress
bending stress resultant multiplied by 6 and divided by the square of the shell thickness (only meaningful for conditions in which the shell is elastic)

Note 1 to entry: The subscript notation for a bending stress relates to the direction of the stress, not the axis about which the shell is deformed.

3.1.4.2
bending stress resultants
bending and twisting moments per unit width of shell that arise as the integral of the first moment of the distribution of direct and shear stresses acting parallel to the shell middle surface through the thickness of the shell, such that under elastic conditions, each of these stress resultants induces a stress state that varies linearly through the shell thickness, with value zero and the middle surface, resulting in two bending moments and one twisting moment at any point (see Figure 3.3b)

Note 1 to entry: Under plastic and partially yielded conditions, the same stress resultants lead to different and often complex stress patterns through the thickness.

3.1.4.3
membrane stress
membrane stress resultant divided by the shell thickness (see Figures 3.2 and 3.3a)

3.1.4.4
membrane stress resultant
force per unit width of the shell wall that arises as the integral of the distribution of direct and shear stresses acting parallel to the shell middle surface through the thickness of the shell

Note 1 to entry: Under elastic conditions, each of these stress resultants induces a stress state that is uniform through the shell thickness, resulting in three membrane stress resultants at any point (see Figures 3.2 and 3.3a).

3.1.4.5
transverse shear stress resultants
forces per unit width of shell that arise as the integral of the distribution of shear stresses acting normal to the shell middle surface through the thickness of the shell, such that under elastic conditions, each of these stress resultants induces a stress state that varies parabolically through the shell thickness, resulting in two transverse shear stress resultants at any point (see Figures 3.2 and 3.3a)

3.1.5 Types of analysis and their use

3.1.5.1
computational analysis
use of shell analysis software (usually finite element) to produce a numerical analysis of the structure

Note 1 to entry: This can take different forms depending on the assumptions adopted in the numerical model.

3.1.5.2
eigenvalue
multiplier on the applied actions that induces a bifurcation

Note 1 to entry: In a computational shell buckling analysis, it is necessary to detect bifurcations from the primary load path. Such possible bifurcations are found using an eigenvalue analysis. The mode of buckling corresponding to an eigenvalue is termed its eigenmode. The term eigenvalue in this standard does not relate to a mode of vibration.

3.1.5.3
geometrically and materially nonlinear analysis
GMNA
computational analysis based on shell bending theory applied to the perfect structure, using the assumptions of nonlinear large deflection theory for the displacements and a fully nonlinear elastic-plastic-hardening material law, where appropriate, and in which a bifurcation eigenvalue check is included at each load level

3.1.5.4
geometrically and materially nonlinear analysis with imperfections explicitly included
GMNIA
computational analysis with imperfections explicitly included, based on the principles of shell bending theory applied to the imperfect structure, including nonlinear large deflection theory for the displacements that accounts fully for any change in geometry due to the actions on the shell and a fully nonlinear elastic-plastic-hardening material law and including a bifurcation eigenvalue check at each load level. The definition of the computational model includes one or more of the following unintended features: deviations of the middle surface from the ideal shape, residual stresses, variations of thickness, misalignment of plates and imperfections in the boundary conditions

3.1.5.5
geometrically nonlinear elastic analysis
GNA
computational analysis based on the principles of shell bending theory applied to the perfect structure, using a linear elastic material law but including nonlinear large deflection theory for the displacements that fully accounts for any change in geometry due to the actions on the shell, including a bifurcation eigenvalue check at each load level

3.1.5.6
geometrically nonlinear elastic analysis with imperfections explicitly included
GNIA
computational analysis with imperfections explicitly included, similar to a GNA analysis as defined in 3.1.5.5, but adopting a model for the geometry of the structure that includes one or more of the following unintended features: deviations of the middle surface from the ideal shape, residual stresses, variations of thickness, misalignment of plates and imperfections in the boundary conditions. It includes a bifurcation eigenvalue check at each load level

3.1.5.7
global analysis
analysis that includes the complete structure, rather than individual structural parts treated separately

Note 1 to entry: This is usually a computational analysis.

3.1.5.8
linear elastic shell analysis
LA
analysis that predicts the behaviour of a thin-walled shell structure on the basis of the small deflection linear elastic shell bending theory, related to the perfect geometry of the middle surface of the shell

Note 1 to entry: It may use standard formulae (see Annex C) or computational analysis.

3.1.5.9
linear elastic bifurcation (eigenvalue) analysis
LBA
analysis that evaluates the linear bifurcation eigenvalue for a thin-walled shell structure on the basis of the small deflection linear elastic shell bending theory, related to the perfect geometry of the middle surface of the shell

Note 1 to entry: It may use standard formulae (see Annexes D and E) or computational analysis.

3.1.5.10
load level
loading condition achieved at the end of each increment in the progressive incrementation of actions until the limit state is reached in a computational nonlinear analysis (GNA, GNIA, MNA, GMNA or GMNIA)

3.1.5.11
materially nonlinear analysis
MNA
analysis based on shell bending theory applied to the perfect structure, using the assumption of small displacement theory, but adopting an ideal elastic-plastic material law (idealised perfectly plastic response after yield) and with no limitation on the plastic strain that can develop

Note 1 to entry: It may use standard formulae (see Annex B) or computational analysis.

3.1.5.12
membrane theory analysis
analysis that predicts the behaviour of a thin-walled shell structure under distributed loads by assuming that only membrane forces satisfy equilibrium with the external loads (see Annex A)

Note 1 to entry: Where global bending of a cylindrical or conical shell is involved without unsymmetrical normal pressures, membrane theory and beam theory lead to the same outcome. Where wind or other unsymmetrical normal pressures are involved, beam theory is no longer valid.

3.1.5.13
semi-membrane theory analysis
analysis that predicts the behaviour of an unsymmetrically loaded or supported thin-walled cylindrical shell structure by assuming that only membrane forces and circumferential bending moments satisfy equilibrium with the external loads

3.1.6 Stress categories used in stress design

3.1.6.1
primary stresses
stress system required for equilibrium with the imposed loading, consisting primarily of membrane stresses, but under some situations bending stresses can also be required to achieve equilibrium

3.1.6.2
secondary stresses
stresses induced by internal compatibility or by compatibility with the boundary conditions, associated with imposed loading or imposed displacements (temperature, pre-stressing, settlement, shrinkage), and not required to achieve equilibrium between an internal stress state and the external loading

3.1.6.3
local stresses
stresses associated with the detailed geometry of constructional details and notches in the shell wall

Note 1 to entry: Relevant constructional details and notches refer to locations such as holes, welds, stepped walls, attachments, stiffener terminations, shell junctions and joints, connections and similar local conditions. These locations lead to local stresses that vary rapidly on a scale smaller than the local thickness of the shell. Such local stresses can generally be ignored in structural resistance evaluations, except those that concern fatigue.

3.1.7 Special definitions for buckling calculations

3.1.7.1
capacity curve
algebraic description of the resistances of all structural systems from elastic imperfect slender systems through elastic-plastic to fully plastic and hardening systems, characterised through the capacity parameters α_G, α_I, β, η_0, η_p, λ_o and χ_h (see 9.5.2 and 9.6.3)

3.1.7.2
characteristic buckling resistance
load associated with buckling in the presence of the geometrical and structural imperfections that are inevitable in practical construction, inelastic material behaviour where appropriate, and follower load effects if relevant (defined in terms of the characteristic values of the modulus and yield stress of the material)

3.1.7.3
characteristic buckling stress
membrane stress or membrane stress component associated with the characteristic buckling resistance

Note 1 to entry: Most stress states in a practical shell involve membrane stresses in different directions at any location, here termed membrane stress components. When buckling is being assessed, it is typical to take only the most significant component in assessing the buckling resistance, sometimes modified by weaker effects of other components.

3.1.7.4
characteristic plastic resistance
load associated with the formation of a complete plastic mechanism generally involving large displacements and extensive strain hardening (defined in terms of the characteristic value of the yield stress of the material). It may be limited by achievement of an acceptable displacement criterion

Note 1 to entry-: The characteristic plastic resistance is usually much larger than the reference plastic resistance since the complete plastic mechanism in a shell involves a complex and spatially varying interaction between membrane and bending stress resultants, together with significant change of geometry.

3.1.7.5
critical buckling resistance
smallest bifurcation load determined assuming the idealised conditions of elastic material behaviour, small deflection theory (no change of geometry), perfect geometry, perfect load application, perfect support, material isotropy and absence of residual stresses (modelled using LBA analysis)

Note 1 to entry: The term “critical” is strictly limited to this meaning alone.

3.1.7.6
critical buckling stress
membrane stress associated with the reference critical buckling resistance

3.1.7.7
design buckling resistance
design value of the buckling load, obtained by dividing the characteristic buckling resistance by the partial factor for resistance

3.1.7.8
design buckling stress
membrane stress or membrane stress component associated with the design buckling resistance

3.1.7.9
design plastic resistance
design value of the plastic resistance, obtained by dividing the characteristic plastic resistance by the partial factor for resistance

3.1.7.10
fabrication tolerance quality class
category of fabrication tolerance requirements that is assumed in design (see 9.4)

3.1.7.11
key value of the stress
value of stress in a non-uniform stress field that is used to characterise the complete pattern of varying stresses in a buckling limit state assessment

3.1.7.12
reference critical buckling resistance
critical buckling resistance used as a reference resistance in the context of the interactions between elastic and plastic behaviour in defining the characteristic resistance of a shell (modelled using LBA analysis)

3.1.7.13
reference plastic resistance
plastic limit load, determined assuming the idealised conditions of rigid-plastic material behaviour, small deflection theory (no change of geometry), perfect geometry, perfect load application, perfect support and material isotropy (modelled using MNA analysis)

3.2 Symbols

For the purposes of this document, the following symbols apply.

3.2.1 Coordinate system

For coordinate system, see Figure 3.1:

r radial coordinate of the shell middle surface, normal to the axis of revolution;

s curvilinear meridional coordinate on general axisymmetric shell;

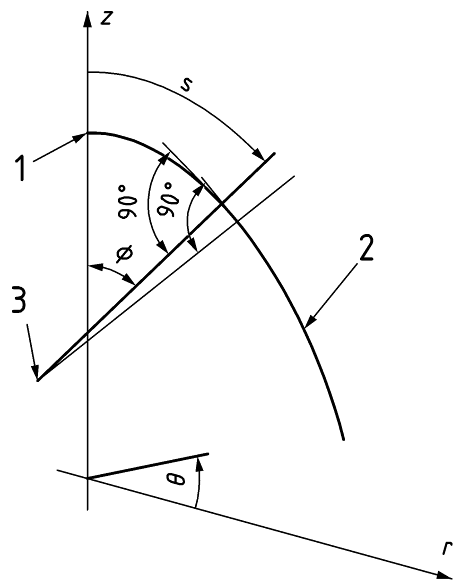
x direction tangential to the meridian of a shell;

NOTE This definition of *x* aligns with the axial direction in a cylindrical shell.

z axial coordinate of a point on the shell middle surface;

θ circumferential coordinate of a point on the shell middle surface;

ϕ meridional slope: angle between axis of revolution and normal to the meridian of the shell.



Key
1 Pole
2 Shell meridian
3 Instantaneous centre of meridional curvature

Figure 3.1 — Coordinate system for a shell of revolution

3.2.2 Shell dimensions

d diameter of the middle surface of a cylindrical shell;

h_i height of shell extending from the upper boundary to the base of course i;

h_{cr} height of the critical buckle in a stepped wall cylinder under external pressure;

L cylinder or cone length between defined boundaries;

ℓ_S length of a shell segment between boundaries that are either BC1 or BC2;

ℓ_g gauge length for measurement of geometric imperfections;

$\ell_{g\theta}$ gauge length in circumferential direction for measurement of geometric imperfections;

ℓ_{gw} gauge length across welds for measurement of geometric imperfections;

ℓ_{gx} gauge length in meridional direction for measurement of geometric imperfections;

ℓ_R boundary zone length in which buckling strength assessment may be omitted (see Annex D, D.4.3);

r radius of the shell middle surface, normal to the axis in an axisymmetric shell;

r_1, r_2 simple radii of the top and bottom of a conical shell;

r_s radius of a spherical shell;

t thickness of shell wall;

t_{max} maximum thickness of shell wall at a joint;

t_{min} minimum thickness of shell wall at a joint;

t_{av} average thickness of shell wall at a joint;

x_e exclusion distance for stress locations involving a tangent modulus reduction (see 5.1);

β apex half angle of cone;

ϕ_0 meridional slope at the support of a spherical shell.

3.2.3 Distributed surface loads and pressures

p_n pressure normal to the shell (outward);

p_x meridional surface loading parallel to the shell;

p_θ circumferential surface loading parallel to the shell;

q pressure normal to the shell (inward).

3.2.4 Line forces

P_n load per unit circumference normal to the shell (outward);

P_x load per unit circumference acting in the meridional direction;

P_θ load per unit circumference acting circumferentially on the shell.

3.2.5 Membrane stress resultants

n_x meridional (axial in a cylinder) membrane stress resultant;

n_θ circumferential membrane stress resultant;

$n_{x\theta}$ membrane shear stress resultant.

3.2.6 Bending stress resultants

m_x meridional (axial in a cylinder) bending moment per unit width;

m_θ circumferential bending moment per unit width;

$m_{x\theta}$ twisting shear moment per unit width;

q_{xn} transverse shear force associated with meridional bending;

$q_{\theta n}$ transverse shear force associated with circumferential bending.

3.2.7 Stresses

σ_x meridional (axial in a cylinder) stress;

σ_θ circumferential stress;

σ_{eq} von Mises equivalent stress (can also take negative values during cyclic loading);

τ, $\tau_{x\theta}$ in-plane shear stress;

τ_{xn}, $\tau_{\theta n}$ meridional, circumferential transverse shear stresses associated with bending.

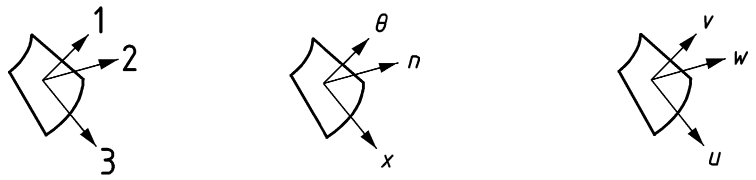
3.2.8 Displacements relative to the perfect or imperfect shell surface

u meridional displacement;

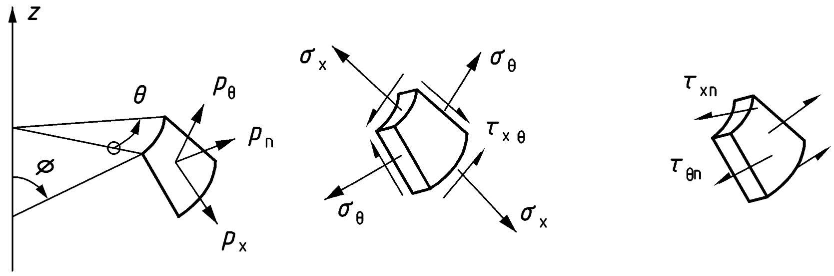
v circumferential displacement;

w displacement normal to the shell surface;

β_ϕ meridional rotation, see 4.3 (4) and 6.2.2.



a) Directions b) Coordinates c) Displacements



d) Surface pressures e) In-plane stresses f) Transverse shear stresses

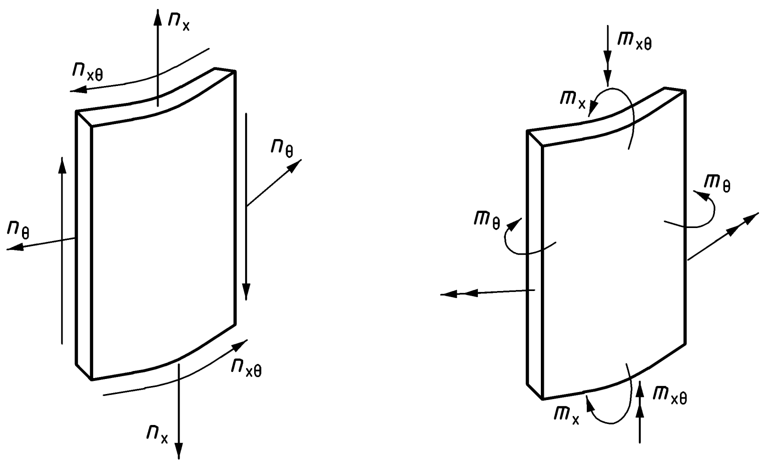
Key

1 Circumferential

2 Normal

3 Meridional

Figure 3.2 — Symbols in a shell of revolution



a) Membrane stress resultants b) Bending stress resultants

Figure 3.3 — Membrane and bending stress resultants in a cylindrical shell

3.2.9 Tolerances

e_a eccentricity between the middle surfaces of joined plates;

U_e unintended eccentricity tolerance parameter;

U_r out-of-roundness tolerance parameter;

U_n initial dimple imperfection amplitude parameter for computational calculations;

U_0 initial dimple tolerance parameter;

δ_a calculated amplitude in a computational treatment (Figure 9.4);

δ_0 tolerance normal to the shell surface;

δ_0 assumed geometric imperfection amplitude;

δ_{u0} meridional interface flatness tolerance between a shell and its support;

δ_{u0} assumed interface flatness amplitude;

δ_{m} measured amplitude using a tolerance measurement (Figure 9.4).

3.2.10 Properties of materials

E Young's modulus of elasticity;

E_{red} reduced elastic modulus to account for stress-strain nonlinearity or thermal effects;

E_{sh} tangent strain hardening modulus;

f_{eq} von Mises equivalent strength;

$f_{\mathrm{p,\theta}}$ temperature-dependent stress-strain proportionality limit;

f_{y} yield strength;

f_{u} ultimate strength;

ν Poisson's ratio.

3.2.11 Parameters in resistance assessment

$a_{\mathrm{p,eq}}$ cyclic plasticity assessment factor;

a_{i} coefficient in buckling strength interaction (see D.4.3);

C_{x} coefficient in axial compression critical buckling resistance;

C_{θ}, $C_{\theta\mathrm{s}}$ coefficients in external pressure critical buckling resistance;

C_{τ}, $C_{\tau\mathrm{s}}$, $C_{\tau\mathrm{L}}$ coefficients in shear critical buckling resistance;

D accumulated fatigue damage (see EN 1993-1-9);

f_{lim} limiting stress for fatigue check (see 6.3.4 (2));

F generalised action;

F_{Ed} action set on a complete structure corresponding to a design situation (design values);

F_{Rd} calculated values of the action set at the maximum resistance condition of the structure (design values);

j_{i} joint efficiency factor, where i = 1 or 2;

k_{f} stress concentration factor in fatigue assessment based on linear analysis;

$k_{\mathrm{f,imp}}$ stress concentration factor in fatigue assessment accounting for local imperfection;

k_{GMNIA} calibration factor on resistance when using nonlinear analyses;

k_{x}, k_{θ}, k_{τ}, k_{i} ratio of i^{th} design stress component to its uniform design buckling stress (see 9.4.2);

k_{ix}, $k_{\mathrm{i\theta}}$, $k_{\mathrm{i\tau}}$ power of interactions in buckling strength interaction (see D.4.3);

$k\mathrm{eq}$ ratio of von Mises equivalent surface stress to equivalent membrane stress at a point;

N_f number of cycles of loading in a fatigue assessment (LS4);

N_{cp} number of cycles of loading in a cyclic plasticity assessment (LS2);

Q fabrication tolerance quality parameter;

R generalised resistance;

R_{cr} reference elastic critical buckling resistance ratio (defined as a load factor on design loads using LBA analysis);

R_k characteristic reference resistance ratio (used with subscripts to identify the basis): defined as a load factor on design loads using the ratio (F_{Rk} / F_{Ed});

R_{pl} reference plastic resistance ratio (defined as a load factor on design loads using MNA analysis);

R_{plf} plastic failure resistance ratio (defined as a load factor on design loads using GMNA analysis);

R_{GNA} buckling resistance ratio determined in a GNA analysis;

R_{GMNA} buckling resistance ratio determined in a GMNA analysis;

R_{GMNIA} buckling resistance ratio determined in a GMNIA analysis (normally as R_k);

R_{MNA} plastic resistance ratio determined in an MNA analysis;

s_{irat} ratio of i^{th} stress state in determining the dominant stress component in buckling;

α elastic buckling reduction factor in buckling strength assessment;

α_G geometric reduction factor;

α_I imperfection reduction factor;

α_{SLM} elastic buckling reduction factor for a segment in the LBA-MNA procedure;

α_S elastic buckling reduction factor for a segment or complete structure;

β plastic range factor in buckling interaction;

γ_F partial factor for actions and action effects;

γ_M partial factor for resistance;

γ_{M0} partial factor for plastic resistance ;

γ_{M1} partial factor for resistance to stability (buckling);

γ_{M2} partial factor for resistance to tensile rupture, including the net section in bolted construction;

γ_{M4} partial factor for resistance to cyclic plasticity;

γ_{Mf} partial factor for resistance to fatigue;

γ_{Ff} partial factor for fatigue loads and load effects;

Δ range of parameter when alternating or cyclic actions are involved;

ε_{mps} maximum permitted von Mises equivalent plastic true strain;

ε_p plastic strain;

$\varepsilon_{p,eq,Ed}$ total accumulated von Mises equivalent plastic strain under cyclic plasticity;

η interaction exponent for buckling;

η_0 value of interaction exponent at $\bar{\lambda} = \bar{\lambda}_0$;

η_p value of interaction exponent at $\bar{\lambda} = \bar{\lambda}_p$;

$\bar{\lambda}$ relative slenderness of a shell;

$\bar{\lambda}_s$ complete shell relative slenderness for a complete shell or shell assembly (multiple segments);

$\bar{\lambda}_{sLM}$ shell segment relative slenderness in the LBA-MNA procedure;

$\bar{\lambda}_0$ squash limit relative slenderness (value of $\bar{\lambda}$, above which resistance reductions due to instability or change of geometry occur);

$\bar{\lambda}_p$ plastic limit relative slenderness (value of below which plasticity affects the stability);

μ hardening exponent for buckling;

μ_0 value of hardening exponent at $\bar{\lambda} = \bar{\lambda}_0$;

μ_h value of hardening exponent at $\bar{\lambda} = 0$;

ψ load combination factor;

ω first relative length parameter for a cylindrical shell;

ω_s upper limit of relative length for short cylindrical shells under external pressure;

Ω second relative length parameter for a cylindrical shell;

ξ third relative length parameter for a cylindrical shell;

χ buckling reduction factor including elastic-plastic effects in buckling strength assessment;

χ_h buckling reduction factor in the hardening zone at $\bar{\lambda} = 0$;

χ_s complete shell buckling reduction factor including elastic-plastic effects in a shell assembly.

3.2.12 Subscripts

E value of stress or displacement (arising from design actions);

F actions;

M material;

R resistance;

cr critical buckling value (see 3.1.7.4);

d design value;

eq von Mises equivalent;

f fatigue;

int internal;

k characteristic value;

max maximum value;

min minimum value;

nom nominal value;

pl plastic value;

plf plastic failure value (LS1);

s for a complete shell, potentially with multiple segments making a shell assembly;

u ultimate;

y yield.

Further symbols are defined where they first occur.

3.3 Sign conventions

(1) Outward direction positive: internal pressure positive, outward displacement positive, except as noted in (5).

(2) Tensile stresses positive, except as noted in (5).

NOTE Compression is treated as positive in EN 1993-1-1.

(3) Shear stresses positive as shown in Figures 3.2 and D.1.

NOTE Although the directions of direct stresses differ between Figures 3.2 and D.1, the direction of in-plane shear is retained unchanged.

(4) Bending moments are defined as positive when they induce tensile stresses on the outer surface of the shell.

(5) For simplicity, in Clause 9 and Annexes D and E, compressive stresses are treated as positive. For these cases, both external pressures and internal pressures are treated as positive where they occur, though the notations p and q are used to identify the direction.

4 Basis of design

4.1 General rules

4.1.1 Basic requirements

(1) The design of shell structures shall be in accordance with the general rules given in EN 1990 and EN 1991 (all parts), and the specific design provisions for steel structures given in the other relevant parts of EN 1993-1 (all parts).

(2) Steel structures designed according to this document shall be executed according to EN 1090-2 with construction materials and products used as specified in the relevant parts of EN 1993, or in the relevant material and product specifications.

(3) This document is intended for use in conjunction with EN 1993-1-1, EN 1993-1-2, EN 1993-1-3, EN 1993-1-4, EN 1993-1-7, EN 1993-1-9, EN 1993-1-14[4] and the relevant application parts of EN 1993, which include:

— Part 3 for towers, masts and chimneys;

— Part 4.1 for silos;

— Part 4.2 for tanks;

— Part 5 for piles.

[4] Under preparation.

4.1.2 Specific requirements

(1) The shell should be designed in such a way that it will sustain all actions and satisfy the following requirements:

— overall equilibrium;

— equilibrium between actions and internal forces and moments, see Clause 7 and Clause 9;

— limitation of cracks due to cyclic plastification, see Clause 8;

— limitation of cracks due to fatigue, see Clause 10.

(2) The design of the shell should satisfy the serviceability requirements in accordance with its intended use and as set out in appropriate application standards (EN 1993-4-1 and EN 1993-4-2).

(3) The shell may be proportioned using design assisted by testing. Where appropriate, the requirements are set out in the appropriate application standard (EN 1993-3, EN 1993-4-1 and EN 1993-4-2).

(4) All actions should be introduced using their design values according to EN 1990, EN 1991 (all parts) and EN 1993-4-1 and EN 1993-4-2 as appropriate.

4.2 Types of analysis

4.2.1 General

(1) One or more of the following types of analysis should be used, depending on the limit state and other considerations:

— Global analysis, see 4.2.2;

— Membrane theory analysis, see 4.2.3;

— Semi-membrane theory analysis, see 4.2.4;

— Linear elastic shell analysis (LA), see 4.2.5;

— Linear elastic bifurcation analysis (LBA), see 4.2.6;

— Geometrically nonlinear elastic analysis (GNA), see 4.2.7;

— Materially nonlinear analysis (MNA), see 4.2.8;

— Geometrically and materially nonlinear analysis (GMNA), see 4.2.9;

— Geometrically nonlinear elastic analysis with imperfections explicitly included (GNIA), see 4.2.10;

— Geometrically and materially nonlinear analysis with imperfections explicitly included (GMNIA), see 4.2.11.

4.2.2 Global analysis

(1) In a global analysis, simplified treatments may be used for discrete parts of the structure consisting of a cylinder, cone, sphere or other structural form, provided that the connections between different parts are appropriately modelled.

4.2.3 Membrane theory analysis

(1) A membrane theory analysis may be used provided that the following conditions are met:

— the boundary conditions are appropriate for transfer of the stresses in the shell into support reactions without causing unacceptable bending effects;

— the shell geometry varies smoothly in shape (without discontinuities);

— the loads have a smooth distribution (without locally concentrated or point loads).

(2) A membrane theory analysis does not meet the requirements of compatibility of deformations at boundaries or between shell segments of different shape or between shell segments subjected to different loading. However, the resulting field of membrane forces satisfies the requirements of equilibrium of the primary stresses (useful for LS1).

NOTE A membrane theory analysis using harmonic series can be useful for unsymmetrical loads with smooth variations (up to cos 4θ) where axial displacements of the boundary are not involved (e.g. see prEN 1991-1-4:—, C.4.4).

4.2.4 Semi-membrane theory analysis

(1) A semi-membrane theory analysis may be used when a long cylindrical shell is subject to a circumferentially varying load with a variation more rapid than a single full cosine around the circumference (i.e. cos θ or 'harmonic 1') and also subject to axial displacements at a boundary (e.g. wind loading with periodic anchors or discretely supported shells).

4.2.5 Linear elastic shell analysis (LA)

(1) The linearity of the theory results from the assumptions of a linear elastic material law and small deformation theory. Small deformation theory implies that the assumed geometry remains that of the undeformed structure. It satisfies compatibility in the deformations as well as equilibrium. The resulting field of membrane and bending stresses satisfy the requirements of primary plus secondary stresses (useful for LS1, LS2, LS3 and LS4).

(2) This analysis is normally undertaken as a computational analysis but in limited cases algebraic formulae may also be used (see Annex C).

(3) Where a computational analysis is undertaken, the modelling, mesh, validation and verification criteria of EN 1993-1-14 should be met.

4.2.6 Linear elastic bifurcation analysis (LBA)

(1) The conditions of 4.2.5 concerning the material and geometric assumptions are met. However, this linear bifurcation analysis obtains the lowest eigenvalue at which the shell can buckle into a different deformation mode, assuming no change of geometry, no change in the direction of action of the loads, and no material degradation. Imperfections of all kinds are ignored. This analysis provides the reference elastic critical buckling resistance R_{cr} (see 9.6, 9.7 and 9.8; useful for LS3), which can be interpreted as a load amplification factor R_{cr} on the design value of the loads F_{Ed}.

(2) This analysis is normally undertaken as a computational analysis but in limited cases algebraic solutions may also be used (see Annexes D and E).

(3) Where a computational analysis is undertaken, the modelling, mesh, validation and verification criteria of EN 1993-1-14 should be met.

(4) This perfect shell elastic critical load should always be determined when the limit state LS3 is verified using GMNIA analysis (see 9.8).

(5) Where this analysis is used as the basis for an LBA-MNA design procedure, multiple eigenvalues (not only the lowest eigenvalue) and their corresponding eigenmodes should be explored to ensure that the eigenmode whose imperfection sensitivity can lead to the lowest elastic buckling prediction is found (see 9.7).

NOTE The imperfection sensitivity of the shell depends on the size and form of the potential buckling mode, so a lower eigenvalue corresponding to an insensitive mode can fail to detect the mode with the lowest imperfect shell buckling resistance.

4.2.7 Geometrically nonlinear elastic analysis (GNA)

(1) A GNA analysis satisfies both equilibrium and compatibility of the deformations under conditions in which the change in the geometry of the structure caused by loading is included. The resulting field of stresses matches the definition of primary plus secondary stresses (useful for LS2 and LS4). This is normally undertaken as a computational analysis.

(2) Where membrane compression or shear stresses are predominant in some part of the shell, a GNA analysis delivers the elastic buckling load of the perfect structure, including changes in geometry, that can be of assistance towards a check of the limit state LS3 (see 9.8).

(3) Where this analysis is used for a buckling load evaluation, the eigenvalues of the system shall be checked throughout the loading path to ensure that the numerical process does not fail to detect a bifurcation in the load path.

(4) The modelling, mesh, validation and verification criteria of EN 1993-1-14 should be met.

4.2.8 Materially nonlinear analysis (MNA)

(1) The result of an MNA analysis leads to the reference plastic limit load, which can be interpreted as a load amplification factor R_{pl} on the design value of the loads F_{Ed}. This analysis provides the reference plastic resistance ratio R_{pl} used in 9.6, 9.7 and 9.8.

(2) This analysis may be undertaken using a computational analysis or algebraic formulae (see Annex B).

(3) Where a computational analysis is undertaken, the modelling, mesh, validation and verification criteria of EN 1993-1-14 should be met.

(4) An MNA analysis may be used to verify limit state LS1.

(5) An MNA analysis may be used to give the plastic strain increment $\Delta\varepsilon_p$ during one cycle of cyclic loading that may be used to verify limit state LS2.

(6) This perfect shell plastic limit load should always be determined when the limit state LS3 is verified using GMNIA analysis (see 9.8).

4.2.9 Geometrically and materially nonlinear analysis (GMNA)

(1) The result of a GMNA analysis, analogously to 4.2.7, gives the geometrically nonlinear plastic failure load of the perfect structure and the plastic strain increment that can be used for checking the limit states LS1 and LS2. This is strictly a computational analysis.

NOTE Where no yielding or plasticity is involved, a GMNA analysis produces the same result as a GNA analysis.

(2) Where compression or shear stresses are predominant in some part of the shell, a GMNA analysis gives the elastic-plastic buckling load of the perfect structure. This perfect shell buckling load should always be determined when the limit state LS3 is verified using GMNIA analysis (see 9.8).

(3) Where this analysis is used for a buckling load evaluation, the eigenvalues of the system should be checked to ensure that the numerical process does not fail to detect a bifurcation in the load path.

(4) The modelling, mesh, validation and verification criteria of EN 1993-1-14 should be met.

4.2.10 Geometrically nonlinear elastic analysis with imperfections explicitly included (GNIA)

(1) A GNIA analysis is used in cases where compression or shear stresses dominate in the shell. It delivers elastic buckling loads of the imperfect structure that can be of assistance in checking the limit states LS3 and LS4 (see 9.8). This is strictly a computational analysis.

(2) Where this analysis is used for a buckling load evaluation (LS3), the eigenvalues of the system should be checked to ensure that the numerical process does not fail to detect a bifurcation in the load path. Care should be taken to ensure that the local stresses do not exceed values at which material nonlinearity can affect the behaviour.

NOTE 1 GNIA analysis is often useful for very thin shells where plasticity plays no role in the ultimate limit state.

NOTE 2 Where a GNIA analysis produces a buckling resistance that is very similar to that of a GNA analysis, the imperfection modelled in the GNIA is not one to which the structure is sensitive.

(3) The modelling, mesh, validation and verification criteria of EN 1993-1-14 should be met, but the imperfections should be defined according to this document.

4.2.11 Geometrically and materially nonlinear analysis with imperfections explicitly included (GMNIA)

(1) A GMNIA analysis is used in cases where compression or shear stresses are dominant in the shell. It delivers elastic-plastic buckling loads for the imperfect structure, that may be used for checking the limit state LS3 (see 9.8). This is strictly a computational analysis.

(2) Where this analysis is used for a buckling load evaluation, the eigenvalues of the system should be checked to ensure that the numerical process does not fail to detect a bifurcation in the load path.

(3) Where this analysis is used for a buckling load evaluation, additional LBA, MNA and GMNA analyses of the perfect shell should always be conducted to ensure that the slenderness is properly recognised and that the degree of imperfection sensitivity of the structural system is identified.

NOTE Whilst not required, it is desirable that an additional GNA analysis is also undertaken to provide the maximum insight into the structural behaviour under nonlinear conditions.

(4) The modelling, mesh, validation and verification criteria of EN 1993-1-14 should be met, but the imperfections should be defined according to this document.

4.3 Shell boundary conditions

(1) The boundary conditions assumed in the design calculation should be chosen in such a way as to ensure that they achieve a realistic or conservative model of the real construction. Special attention should be given not only to the constraint of displacements normal to the shell wall (deflections), but also to whether the displacements in the plane of the shell wall (meridional and circumferential) are adequately constrained because of the significant effect these displacements can have on the shell strength and buckling resistance.

NOTE The buckling resistance of a shell is often sensitive to any minor flexibility in the boundary conditions, making the modelling of realistic boundary conditions more critical than for a simple load-deformation analysis. Boundary conditions relating to membrane displacements in the shell often have a strong influence on buckling resistances.

(2) In shell buckling (eigenvalue) calculations (limit state LS3), the definition of the boundary conditions should refer to the incremental displacements during the buckling process, and not to total displacements induced by the applied actions before buckling.

(3) The boundary conditions at a continuously supported lower edge of a shell should take into account whether local uplifting of the shell is fully prevented or not.

(4) The shell edge rotation β_{ϕ} should be particularly considered in short shells and in the calculation of secondary stresses in longer shells (according to the limit states LS2 and LS4).

(5) The boundary conditions set out in 6.2.2.2 should be used in computer analyses and in selecting formulae from Annexes A to E.

(6) The structural connections between shell segments at a junction should be such as to ensure that the boundary condition assumptions used in the design of the individual shell segments are satisfied.

4.4 Verification by the partial factor method

(1) Where structural properties are determined by testing, the requirements and procedures of EN 1990 should be adopted.

(2) The partial factors γ_{Mi} for different limit states should be taken from Table 4.1.

Table 4.1 — Partial factors for resistance

Resistance to failure mode	Relevant γ
Resistance of welded or bolted shell wall to plastic limit state	γ_{M0}
Resistance of shell to stability	γ_{M1}
Resistance of welded or bolted shell wall to rupture	γ_{M2}
Resistance of shell to cyclic plasticity	γ_{M4}
Resistance of shell to fatigue	γ_{Ff}

(3) The numerical values in Table 4.2 are recommended for shell structures that are not covered by the provisions of EN 1993-4-1 or EN 1993-4-2.

Table 4.2 (NDP) — Numerical values for partial factors for resistance for shell structures outside the scope of EN 1993-3, EN 1993-4-1 and EN 1993-4-2

γ_{M0} = 1,00	γ_{M1} = 1,10	γ_{M2} = 1,25
γ_{M4} = 1,00		see EN 1993-1-9

NOTE 1 The values of the partial factors γ_{Mi} are given in Table 4.2 (NDP) unless the National Annex gives different values.

NOTE 2 When a reliability analysis is used to determine the appropriate partial factor for shell buckling γ_{M1}, it is found to depend quite strongly on the structural form, the slenderness of the shell, the load case and the buckling mode, since the imperfection sensitivity and the consequent variability of the buckling resistance varies considerably with these factors. Due to lack of reliable data relevant to practical construction, the drafting committee chose to retain the historically accepted value of γ_{M1}.

5 Materials and geometry

5.1 Material properties

(1) The relevant material properties of carbon steels should be obtained from the relevant application standard.

(2) For the mechanical properties of the structural carbon steels S235, S275, S355, S420 and S460 and also for weathering steel grades S235, S275 and S355, see EN 1993-1-1, except as defined in (3).

(3) For all the steels covered by this document, the design value of Poisson's ratio should be taken as ν = 0,3. The characteristic value of the elastic modulus for structural steel should be taken as E = 200 000 N/mm^2, in accordance with the value defined for stability calculations in EN 1993-1-14.

NOTE Most of the design rules in this document have been derived from GMNIA calculations and not from experiments. The value of elastic modulus defined in EN 1993-1-14 provides a safe choice where it is required to define the elastic buckling resistance, which is the controlling resistance for most thin shells. This value is lower than that given in EN 1993-1-1, where serviceability is its primary role. The mismatch between these two values is unimportant since the value used here is a conservative choice. The same choice is also relevant to stainless steels.

(4) For stainless steels covered by this document, the characteristic value of the elastic modulus should be taken as E = 191 000 N/mm^2, in accordance with the value defined for stability calculations in EN 1993-1-14.

(5) Other relevant material properties of stainless steels should be obtained from EN 1993-1-4.

(6) In a computational analysis using materials with a nonlinear stress-strain relationship, the 0,2% proof stress should be used to represent the yield stress f_y in all relevant formulae. The stress-strain curve should be modelled in accordance with the requirements of prEN 1993-1-14:—, 5.3.

(7) Where a material with a nonlinear stress-strain curve is involved and a buckling analysis is carried out under stress design (see 9.5) and the special provisions for stainless steel do not apply, the initial tangent value of Young´s modulus E should be replaced by a reduced value E_{red}. If no better method is available, the linear elastic stress state should be examined. The peak value of the von Mises equivalent stress derived from the membrane stress components alone at any point in the structure that is more distant than x_e from any boundary should be found. The tangent modulus (from a tensile test) corresponding to this stress should then be taken as E_{red} to replace the elastic modulus E and thus to obtain an estimate of the quasi-elastic critical load or quasi-elastic critical stress.

(8) The exclusion distance x_e may be taken as equal to the boundary zone distance ℓ_R defined in Annex D.4.3.

(9) The provisions of this standard are relevant to material properties at temperatures not exceeding 100 °C, except as defined in (10).

(10) The variation of properties of steels at temperatures above 100 °C are given in EN 1993-1-2. Where these are adopted, the reduced yield stress may be conservatively taken as the temperature-dependent proportionality limit $f_{\mathrm{p},\theta}$ defined therein. The mechanical properties of steel grades not represented in EN 1993-1-2 should be based on reliable information.

(11) High strength steels, as defined by EN 1993-1-12, should only be used with a GMNA or GMNIA analysis for the purposes of verifying LS1, LS2 and LS3.

5.2 Design values of geometrical data

(1) The thickness t of the shell should be taken as defined in the relevant application standard. If no application standard is relevant, the nominal thickness of the wall, reduced by the prescribed value of the corrosion loss and ignoring any coatings, should be used.

(2) The thickness ranges within which the rules of this standard may be applied are defined in the relevant EN 1993 application parts.

(3) The middle surface of the shell should be taken as the reference surface for loads.

(4) The simple radius r of the shell should be taken as the nominal radius of the middle surface of the shell, measured normal to the axis of revolution.

NOTE This radius varies with position on the axis in all shells that are not simply cylindrical.

(5) The buckling design rules of this standard should not be applied outside the ranges of the r/t ratio set out in 1.1 (12), or where stricter restrictions apply as defined in 9, 10, Annex D or E, or in the relevant EN 1993 application parts.

5.3 Geometrical tolerances and geometrical imperfections

(1) Tolerance values for the deviations of the geometry of the shell surface from the nominal values are defined in the execution standard EN 1090-2. Relevant categories for the design of shells for the ultimate limit state of buckling (LS3) (see 9.4) are:

— local dimples (local normal deviations from the nominal middle surface);

— out-of-roundness (deviation from circularity);

— eccentricities (deviations from a continuous middle surface in the direction normal to the shell across the junctions between plates);

— deviations of the base of a shell from full contact with the support.

NOTE The requirements for execution are set out in EN 1090-2, but a fuller description of these tolerances is given here because there is a critical relationship between the form of the tolerance measure, its amplitude and the evaluated buckling resistance of a shell. The buckling-relevant tolerances defined here can differ from those defined in EN 1090-2.

(2) If the limit state of buckling (LS3, as described in 6.3.3) is one of the ultimate limit states to be considered, the buckling-relevant geometrical tolerances should be carefully observed in order to keep the geometrical imperfections within specified limits. These buckling-relevant geometrical tolerances and the conditions to which they are relevant are identified and quantified in 9 or in the relevant EN 1993 application parts.

(3) Calculation values for the deviations of the shell surface geometry from the nominal geometry, as required for geometrical imperfection assumptions (complete shell imperfections or local imperfections) for buckling design by computational GMNIA analysis, should be derived from the specified geometrical tolerances (see 9.4). Relevant rules are given in 9.8 or in relevant EN 1993 application parts.

(4) If the limit state of fatigue (LS4, as described in 6.3.4) is one of the ultimate limit states to be considered, consideration should be given to adopting an appropriate choice of the buckling-relevant geometrical tolerances as imposed imperfections that may exacerbate fatigue failure at a specific location (see 9.4).

6 Structural analysis

6.1 Types of design

6.1.1 Stress design

6.1.1.1 General

(1) Where the stress design approach is used, the limit states should be assessed in terms of three categories of stress: primary, secondary and local. The categorisation is performed, in general, on the von Mises equivalent stress at a point, but buckling stresses cannot be assessed reliably using this value.

6.1.1.2 Primary stresses

(1) The primary stresses should be taken as the stress system required for equilibrium with the imposed loading. They may be calculated from any realistic statically admissible determinate system. The plastic failure limit state (LS1) should be deemed to be reached when the primary stress reaches the yield strength throughout the full thickness of the wall at a sufficient number of points, such that only the strain hardening reserve or a change of geometry would lead to an increase in the resistance of the structure.

(2) The calculation of primary stresses should be based on any system of stress resultants, consistent with the requirements of equilibrium of the structure. It may also take into account the benefits of plasticity theory. Alternatively, since linear elastic analysis satisfies equilibrium requirements, its predictions may also be used as a conservative representation of the plastic failure limit state (LS1). Any of the analysis methods given in 4.2.3, 4.2.5 and 6.2.3 may be applied.

(3) Because limit state design for LS1 allows for full plastification of the cross-section, the primary stresses due to bending moments may be calculated on the basis of the plastic section modulus, see 7.2.1. Where there is interaction between stress resultants in the cross-section, interaction rules based on the Ilyushin yield criterion may be applied.

(4) The primary stresses should be limited to the design value of the yield strength, see Clause 7 (LS1).

6.1.1.3 Secondary stresses

(1) In statically indeterminate structures, account should be taken of the secondary stresses, induced by internal compatibility and compatibility with the boundary conditions, that are caused by imposed loading or imposed displacements (temperature, prestressing, settlement, shrinkage).

NOTE As the von Mises yield condition is approached, the local strains in the structure increase without further increase in the stress state.

(2) Where cyclic loading causes plasticity, and several loading cycles occur, consideration should be given to the possible reduction of resistance caused by the secondary stresses. Where the cyclic loading is of such a magnitude that yielding occurs both at the maximum load and again on unloading, account should be taken of a possible failure by cyclic plasticity associated with the secondary stresses (LS2).

(3) If the stress calculation is carried out using a linear elastic analysis that allows for all relevant compatibility conditions (effects at boundaries, junctions, variations in wall thickness, misalignment of the middle surface etc.), the stresses that vary linearly through the thickness may be taken as the sum of the primary and secondary stresses and used in an assessment involving the von Mises yield criterion, see 7.2.

NOTE The secondary stresses are never needed in an evaluation without inclusion of the primary stresses.

(4) The secondary stresses should be limited as follows:

— The sum of the cyclic change in the von Mises equivalent surface stress derived from the sum of the primary and secondary stresses (including bending stresses) should be limited to $2f_{yd}$ for the condition of cyclic plasticity, see Clause 8 (LS2);

— The membrane component of the sum of the primary and secondary stresses should be limited by the design buckling resistance, see Clause 9 (LS3).

— The sum of the cyclic change in the surface maximum principal stress derived from the primary and secondary stresses (including bending stresses) should be limited to the nominal fatigue resistance, see Clause 10 (LS4) and EN 1993-1-9.

6.1.1.4 Local stresses

(1) The highly localised stresses associated with stress raisers in the shell wall due to notch effects (holes, welds, stepped walls, attachments, stiffener terminations, shell junctions and joints) should be taken into account in a fatigue assessment (LS4) using the provisions for modified nominal stresses defined in EN 1993-1-9.

(2) For construction details given in EN 1993-1-9, the fatigue design may be based on the nominal linear elastic stresses (sum of the primary and secondary stresses) at the relevant point. For all other details, the local stresses may be calculated by applying stress concentration factors (notch factors) according to EN 1993-1-9 to the stresses calculated using a linear elastic analysis.

(3) The local stresses should be limited according to the requirements for fatigue (LS4) set out in Clause 10 and EN 1993-1-9.

6.1.2 Design using standard formulae

(1) Where this concept is used, the limit states may be represented by standard formulae that have been derived from either membrane theory, plastic mechanism theory, linear elastic analysis or geometrically and materially nonlinear analysis with explicit imperfections.

(2) The membrane theory formulae given in Annex A may be used to determine the primary stresses needed for assessing LS1 and LS3.

(3) The formulae for plastic design given in Annex B may be used to determine the reference plastic resistances needed for assessing LS1.

(4) The formulae for linear elastic analysis given in Annex C may be used to determine stresses of the primary plus secondary stress type needed for assessing LS2 and LS4. An LS3 assessment may be based on the membrane part of these formulae.

(5) The formulae given in Annex D may be used to give direct assessment of the design buckling resistance of cylindrical shells under uniform loads according to LS3.

(6) The formulae for reference resistance design given in Annex E may be used to give direct assessment of the design buckling resistance for assessing LS3. Where the formulae of Annex E are used with the imperfection amplitude δ_0 assigned to be zero, Annex E may also be used to assess LS1.

6.1.3 Design by computational analysis

(1) Where a computational analysis is used, the assessment of the limit states should be carried out using one of the alternative types of analysis specified in 4.2 (but not membrane or semi-membrane theory analysis) applied to the complete structure.

(2) Linear elastic analysis (LA) may be used to determine stresses or stress resultants, for use in assessing LS2 and LS4. The membrane parts of the stresses found by LA may be used in assessing

LS3. LS1 may be assessed using LA, but LA only gives an approximate and safe estimate and its results should be interpreted as set out in Clause 7.

(3) Linear elastic bifurcation analysis (LBA) may be used to determine the reference elastic critical buckling resistance of the structure, for use in assessing LS3.

(4) A materially nonlinear analysis (MNA) may be used to determine the reference plastic resistance, and this may be used for assessing LS1. Under a cyclic loading history, an MNA analysis may be used to determine plastic strain incremental changes, for use in assessing LS2. The reference plastic resistance is also required as part of the assessment of LS3, and this may be found from an MNA analysis.

(5) Geometrically nonlinear elastic analyses (GNA and GNIA) include consideration of the deformations of the structure, but none of the design methodologies of 9 (LS3) permit these to be used without a GMNIA analysis. A GNA analysis may be used to determine the elastic buckling load of the perfect structure. A GNIA analysis may be used to determine the elastic buckling load of the imperfect structure. A GNIA analysis, with an appropriate choice of geometrical imperfections, may also be used to determine the stresses relevant to a fatigue assessment (LS4).

(6) Geometrically and materially nonlinear analysis (GMNA and GMNIA) may be used to determine collapse loads for the perfect (GMNA) and the imperfect structure (GMNIA). The GMNA analysis may be used in assessing LS1, as detailed in 7.3. The GMNIA collapse load may be used, with additional consideration of the results of LBA, MNA and GMNA analyses to assess LS3 as detailed in 9.8. Under a cyclic loading history, the plastic strain incremental changes taken from a GMNA analysis may be used for assessing LS2, as detailed in 8.3.

6.2 Stress resultants and stresses in shells

6.2.1 Stress resultants in the shell

(1) In principle, the eight stress resultants in the shell wall at any point should be calculated and the assessment of the shell with respect to each limit state should take all of them into account. However, the through thickness transverse shear stresses τ_{xn}, $\tau_{\theta n}$ due to the transverse shear forces q_{xn}, $q_{\theta n}$ are insignificant compared with the other components of stress in almost all practical cases, so they can usually be neglected.

(2) Accordingly, for most purposes, the evaluation of the limit states may be made using only the six stress resultants in the shell wall n_x, n_θ, $n_{x\theta}$, m_x, m_θ, $m_{x\theta}$. Where the structure is axisymmetric and subject only to axisymmetric loading and support, only n_x, n_θ, m_x and m_θ need be used.

(3) If any uncertainty arises concerning the stress to be used in any of the limit state verifications, the von Mises equivalent stress on the shell surface may be used as a safe estimate.

6.2.2 Modelling of the shell for analysis

6.2.2.1 Geometry

(1) The shell should be represented by its middle surface.

(2) The radius of curvature should be taken as the nominal radius of curvature. Imperfections should be neglected, except as set out in Clause 9 (LS3 buckling limit state) and Clause 10 (LS4 fatigue limit state).

(3) An assembly of shell segments should not be subdivided into separate segments for analysis unless the boundary conditions for each segment are chosen in such a way as to represent interactions between them in a conservative manner.

(4) A base ring intended to transfer local support forces into the shell should not be separated from the shell it supports in an assessment of limit state LS3.

(5) Eccentricities and steps in the shell middle surface should be included in the analysis model if they induce significant bending effects as a result of the membrane stress resultants following an eccentric path.

(6) At junctions between shell segments, any eccentricity between the middle surfaces of the shell segments should be considered in the modelling.

(7) A ring stiffener should be treated as a separate discrete structural component of the shell, except where the spacing of the rings is closer than $1{,}5\sqrt{rt}$ in which case a smeared orthotropic shell model may be used.

(8) A shell that has discrete stringer stiffeners attached to it may be treated as an orthotropic uniform shell, provided that the stringer stiffeners are no further apart than $5\sqrt{rt}$.

NOTE For a fuller treatment of discrete stringer stiffeners, see EN 1993-4-1.

(9) A shell that is corrugated (vertically or horizontally) may be treated as an orthotropic uniform shell provided that the corrugation full wavelength is less than $0{,}5\sqrt{rt}$ where t is the local plate thickness.

NOTE For a fuller treatment of corrugated shells, see EN 1993-4-1.

(10) A hole in the shell may be neglected in the modelling provided its largest dimension is smaller than $0{,}6\sqrt{rt}$.

(11) The overall stability of the complete structure should be verified as detailed in EN 1993-3, EN 1993-4-1 and EN 1993-4-2, as appropriate.

6.2.2.2 Boundary conditions

(1) The appropriate boundary conditions should be used in analyses for the assessment of limit states in shell segments using the boundary conditions defined in Table 6.1. For the special conditions needed for buckling calculations, see 9.3.

(2) Rotational restraints at shell boundaries may be neglected in modelling for limit state LS1, but should be included in modelling for limit states LS2 and LS4. For short shells, as classified in Annexes D and E, any boundary rotational restraint should be included for limit state LS3.

(3) Support boundary conditions should be checked to ensure that they do not cause excessive non-uniformity of transmitted forces or introduced forces that are eccentric to the shell middle surface. The provisions of the relevant EN 1993 application parts should be adopted when applying this rule to silos, tanks, chimneys and towers.

(4) When a computational analysis is used, the boundary condition for the normal displacement w should also be used for the circumferential displacement v, except where special circumstances make this inappropriate.

Table 6.1 — Boundary conditions for shell segments

Boundary condition code	Simple term	Displacements	Displacements normal to the shell surface	Meridional or axial displacements	Meridional rotation
BC1r	Clamped	radial fixed meridional fixed rotation fixed	$w = 0$	$u = 0$	$\beta_\phi = 0$
BC1f		radial fixed meridional fixed rotation free	$w = 0$	$u = 0$	$\beta_\phi \neq 0$
BC2r		radial fixed meridional free rotation fixed	$w = 0$	$u \neq 0$	$\beta_\phi = 0$
BC2f	Pinned	radial fixed meridional free rotation free	$w = 0$	$u \neq 0$	$\beta_\phi \neq 0$
BC2s		radial elastically restrained meridional free rotation free	w elastically restrained by a “stiff member” or “spring”	$u \neq 0$	$\beta_\phi \neq 0$
BC2u		radial fixed meridional elastically restrained rotation free	$w = 0$	u elastically restrained by a “stiff support” or “spring”	$\beta_\phi \neq 0$
BC3r		radially free meridionally free rotation fixed	$w \neq 0$	$u \neq 0$	$\beta_\phi = 0$
BC3f	Free edge	radial free meridional free rotation free	$w \neq 0$	$u \neq 0$	$\beta_\phi \neq 0$

The circumferential displacement v is closely linked to the displacement w normal to the surface, so separate boundary conditions are not identified for these two parameters. Instead, the values in the column “Displacements normal to the shell surface” should be adopted for displacement v.

NOTE The required stiffness of the boundary ring to achieve a BC2f condition (rather than an enhanced BC3) under external pressure is defined in D.5.3.4.

6.2.2.3 Actions and environmental influences

(1) Actions should all be assumed to act at the shell middle surface. Eccentricities of load should be represented by static equivalent forces and moments at the shell middle surface.

(2) Local actions and local patches of action should not be represented by equivalent uniform loads.

(3) The modelling should account for whichever of the following are relevant:

— residual stresses or deformations arising from the construction process;

— local settlement under shell walls;

— local settlement under discrete supports;

— uniformity / non-uniformity of support of the structure;

— thermal differentials from one side of the structure to the other;

— thermal differentials from inside to outside of the structure;

— wind effects on openings and penetrations;

— interaction of wind effects on groups of structures;

— connections to other structures;

— conditions during erection.

NOTE Significant residual forces can arise due to progressive construction processes (e.g. on rolled plate assemblies used in on-site construction).

6.2.3 Types of analysis

(1) The design should be based on one or more of the types of analysis given in Table 6.2. The conditions defined in 4.2 should be adopted when using each type of analysis.

Table 6.2 — Types of shell analysis

Type of analysis	**Shell theory**	**Material law**	**Shell geometry**
Membrane theory of shells	membrane equilibrium	not applicable	perfect
Semi-membrane theory of shells	linear circumferential bending, membrane shear and axial stretching	linear	perfect
Linear elastic shell analysis (LA)	linear bending and stretching	linear	perfect
Linear elastic bifurcation analysis (LBA)	linear bending and stretching	linear	perfect
Geometrically nonlinear elastic analysis (GNA)	nonlinear	linear	perfect
Materially nonlinear analysis (MNA)	linear	ideal elastic-plastic ($E_{sh} < 10^{-3}E$)	perfect
Geometrically and materially nonlinear analysis (GMNA)	nonlinear	nonlinear	perfect
Geometrically nonlinear elastic analysis with imperfections explicitly included (GNIA)	nonlinear	linear	imperfect
Geometrically and materially nonlinear analysis with imperfections explicitly included (GMNIA)	nonlinear	nonlinear	imperfect

6.3 Ultimate limit states to be considered

6.3.1 LS1: Plastic failure

(1) The limit state of the plastic failure should be taken as the condition in which the capacity of the structure to resist the actions on it is exhausted by plasticity in the material.

NOTE The plastic failure resistance differs from the reference plastic resistance. The reference plastic resistance is strictly found as the plastic collapse load of the perfect structure obtained from a mechanism based on small displacement theory using an ideal elastic-plastic material law (MNA).

(2) The limit state of tensile rupture should be taken as the condition in which the shell wall experiences gross section tensile failure, leading to separation of the two parts of the shell.

(3) In the absence of fastener holes, verification at the limit state of tensile rupture may be assumed to be covered by the check for the plastic failure limit state. However, where holes for fasteners occur, a supplementary check in accordance with EN 1993-1-1 or EN 1993-1-8 should be carried out.

(4) In verifying the plastic failure limit state, plastic or partially plastic behaviour of the structure may be assumed (i.e. elastic compatibility considerations may be neglected).

NOTE 1 Since the plastic failure limit state includes change of geometry, this limit state can also capture snap-through buckling, which can occur in the elastic state. The reference plastic resistance does not include changes of geometry, so this apparent anomaly does not occur.

NOTE 2 The plastic failure limit state does not include considerations of bifurcation, so no checks for bifurcation are required when a GMNA analysis is used to assess the plastic failure limit state LS1.

(5) All relevant load combinations should be accounted for when checking LS1.

(6) One or more of the following methods of analysis (see 4.2) should be used for the calculation of the design stresses and stress resultants when checking LS1:

— membrane theory;

— formulae in Annexes A and B;

— linear elastic analysis (LA);

— materially nonlinear analysis (MNA);

— geometrically and materially nonlinear analysis (GMNA).

6.3.2 LS2: Cyclic plasticity

(1) The limit state of cyclic plasticity should be taken as the condition in which repeated cycles of loading and unloading produce repeated yielding in tension and in compression at the same point, thus causing plastic work to be repeatedly done on the structure, eventually leading to local cracking by exhaustion of the energy absorption capacity of the material.

NOTE The stresses that are associated with this limit state develop under a combination of all actions and the compatibility conditions for the structure.

(2) All variable actions (such as imposed loads and temperature variations) that can lead to yielding, and which might be applied with more than three cycles in the life of the structure, should be considered when checking LS2.

(3) Where the number of cycles involving cyclic plasticity is greater than N_{cp}, the provisions of LS2 should be used.

NOTE The value of N_{cp} is taken as N_{cp} = 20, unless the National Annex gives a different value.

(4) In the verification of this limit state, compatibility of the deformations under elastic or elastic-plastic conditions should be considered.

(5) One or more of the following methods of analysis (see 4.2) should be used for the calculation of the design stresses and stress resultants when checking LS2:

— formulae in Annex C;

— elastic analysis (LA or GNA);

— MNA or GMNA to determine the plastic strain range.

(6) Low cycle fatigue failure may be assumed to be prevented if the procedures set out in this standard are adopted.

6.3.3 LS3: Buckling

(1) The limit state of buckling should be taken as the condition in which all or part of the structure suddenly develops large displacements normal to the shell surface, caused by loss of stability under compressive membrane or shear membrane stresses in the shell wall, leading to inability to sustain any increase in the stress resultants, and possibly causing total collapse of the structure.

(2) One or more of the following methods of analysis (see 4.2) and buckling resistance assessment should be used for the calculation of the design stresses and stress resultants when checking LS3:

— membrane theory for axisymmetric loading and global bending conditions only (for exceptions, see the relevant application parts EN 1993-3, EN 1993-4-1 and EN 1993-4-2;

— formulae in Annexes A, D and E;

— reference resistance design, where the formulae in Annex E refer to the specific geometry, loading and boundary conditions of the structure;

— linear elastic analysis (LA), which is a minimum requirement for stress analysis under general loading conditions with formulae in Annex D (except where the stress analysis of the load case is given in Annex A, or where the buckling condition is treated as a special case in Annex D);

— linear elastic bifurcation analysis (LBA), which is required for shells under general loading conditions if the critical buckling resistance is to be used in an LBA-MNA assessment, or a GMNIA assessment;

— materially nonlinear analysis (MNA), which is required for shells under general loading conditions if the true reference plastic resistance (rather than a lower bound estimate taken from an LA analysis) is to be used in an LBA-MNA assessment;

— GMNIA, together with supporting MNA, LBA and GMNA analyses, and using appropriate imperfections and calculated calibration factors.

(3) All relevant load combinations causing compressive membrane or shear membrane stresses in the shell should be accounted for when checking LS3.

(4) Because the strength under limit state LS3 depends strongly on the quality of construction, the strength assessment should take account of the associated requirements for execution tolerances. For this purpose, three classes of geometrical tolerances, termed “fabrication quality classes” are given in 9.

6.3.4 LS4: Fatigue

(1) The limit state of high cycle fatigue should be taken as the condition in which repeated cycles of increasing and decreasing stress caused by variable actions lead to the development and propagation of a fatigue crack.

(2) A fatigue verification according to Clause 10 and EN 1993-1-9 should be carried out for shell structures exposed to high cycle variable actions. However, this verification may be omitted provided that, in the design life of the structure according to the relevant action spectrum defined in EN 1991 and the appropriate application parts of EN 1993-3, the following two criteria are both met:

— the design value of the peak von Mises equivalent surface stress at all points in the structure calculated using an LA analysis is less than f_{lim};

— no variable load is applied with more than N_{f} cycles.

NOTE The value of the peak stress $f_{\lim}$ is taken as 150 N/mm^2 and N_f is taken as 10 000 unless the National Annex gives different values.

(3) The appropriate method of analysis (see 4.2) among the following should be used for the calculation of the design stresses and stress resultants for use with the provisions of EN 1993-1-9 when checking LS4:

— analysis using membrane theory (beam theory) for long, thick-walled cylinders;

— formulae in Annex C;

— linear elastic analysis (LA);

— nonlinear elastic analysis (GNA)

— nonlinear imperfect elastic analysis (GNIA).

(4) Additional stress concentration factors k_f and $k_{f,imp}$ may be needed to account for eccentricities, imperfections and other global stress-raising effects that are not included in the analysis calculation and are not specifically addressed by the fatigue classes of EN 1993-1-9. Further information on the relationship between the choice of analysis method, the applicable stress concentration factors k_f and $k_{f,imp}$ and the verification procedure of EN 1993-1-9 are given in Clause 10.

(5) Where the number of cycles involved in the assessment is less than N_f, the provisions for LS2 (Clause 8) should be adopted.

6.4 Concepts for the limit state verifications

6.4.1 General

(1) The limit state verification should be carried out using one of the following:

— stress design;

— standard formulae;

— computational analysis.

(2) Account should be taken of the fact that elastic-plastic material responses induced by different stress components in the shell have different effects on the failure modes and the ultimate limit states. The stress components should therefore be placed in stress categories with different limits. Stresses that develop to meet equilibrium requirements should be treated as more significant than stresses that are induced by the compatibility of deformations normal to the shell. Local stresses caused by notch effects in construction details may be assumed to have a negligibly small influence on the resistance to static loading.

(3) The categories distinguished in stress design should be primary, secondary and local stresses. Primary and secondary stress states may be replaced by stress resultants where appropriate.

(4) When using a computational analysis, the primary and secondary stress states should be replaced by the limit load and the strain range for cyclic loading.

(5) In general, it may be assumed that primary stress states control LS1, LS3 depends strongly on primary stress states but can be affected by yielding caused by secondary stress states, LS2 depends on the combination of primary and secondary stress states, and local surface stresses govern LS4.

7 Plastic failure Limit State (LS1)

7.1 Design values of actions

(1) The design values of the actions shall be based on the most adverse relevant load combination (including the relevant γ_F and ψ factors).

(2) Only those actions that represent loads affecting the equilibrium of the structure need be included.

7.2 Stress design

7.2.1 Design values of stresses

(1) Although stress design is based on an elastic analysis and therefore cannot accurately predict the plastic failure limit state, it may be used, on the basis of the lower bound theorem, to provide a conservative assessment of the plastic collapse resistance which is used to represent the plastic failure limit state, see 6.1.1.

(2) The Ilyushin yield criterion may be used, as detailed in (6), as it comes closer to the true plastic collapse state than a simple elastic surface stress evaluation.

(3) At each point in the structure the design value of the von Mises equivalent stress $\sigma_{eq,Ed}$ should be taken as the highest primary stress determined in a structural analysis that considers the laws of equilibrium between imposed design load and internal forces and moments.

(4) The primary stress may be taken as the maximum value of the stresses required for equilibrium with the applied loads at a point or along an axisymmetric line in the shell structure.

(5) Where a membrane theory analysis is used, or where a linear bending theory analysis (LA) is used subject to the conditions defined in (6), the resulting two-dimensional field of stress resultants $n_{x,Ed}$, $n_{\theta,Ed}$ and $n_{x\theta,Ed}$ may be represented by the von Mises equivalent design stress $\sigma_{eq,Ed}$ obtained from:

$$\sigma_{eq,Ed} = \frac{1}{t}\sqrt{n_{x,Ed}^2 + n_{\theta,Ed}^2 - n_{x,Ed} \cdot n_{\theta,Ed} + 3n_{x\theta,Ed}^2} \tag{7.1}$$

NOTE Where an LA or GNA analysis is used, it is possible that the computational tool will give only surface stresses. For each membrane stress component, the corresponding membrane stress resultant *n* can be found from the mean of the values on the two surfaces multiplied by the thickness at that location. The bending moment component can similarly be found from the difference between the two surface values (see 3.1.4.1 to 3.1.4.4).

(6) Where an LA or GNA analysis is used, the magnitude of the largest von Mises equivalent surface stress found using Formulae (7.2) to (7.5) should be evaluated and compared with the von Mises equivalent membrane stress found using Formula (7.1) at the same location from the same analysis.

(7) Where the largest von Mises equivalent surface stress exceeds k_{eq} times the value from Formula (7.1), the von Mises equivalent stress should be taken as the value determined using Formulae (7.2) to (7.5).

(8) The value of k_{eq} should be taken as k_{eq} = 3, unless a better value can be obtained by rational analysis relevant to the specific condition.

$$\sigma_{eq,Ed} = \sqrt{\sigma_{x,Ed}^2 + \sigma_{\theta,Ed}^2 - \sigma_{x,Ed} \cdot \sigma_{\theta,Ed} + 3\tau_{x\theta,Ed}^2} \tag{7.2}$$

in which

$$\sigma_{x,Ed} = \frac{n_{x,Ed}}{t} \pm \frac{m_{x,Ed}}{(t^2/4)}, \tag{7.3}$$

$$\sigma_{\theta,\mathrm{Ed}} = \frac{n_{\theta,\mathrm{Ed}}}{t} \pm \frac{m_{\theta,\mathrm{Ed}}}{(t^2/4)} \tag{7.4}$$

$$\tau_{x\theta,\mathrm{Ed}} = \frac{n_{x\theta,\mathrm{Ed}}}{t} \pm \frac{m_{x\theta,\mathrm{Ed}}}{(t^2/4)} \tag{7.5}$$

NOTE Formulae (7.2) to (7.5) provide a simplified conservative equivalent stress for design purposes.

7.2.2 Design values of resistance

(1) The potential for failure by rupture under membrane stress components should be evaluated separately from that for failure by surface yielding.

(2) The membrane components of the stresses should be evaluated using Formula (7.1). For this evaluated stress, the design value of the von Mises equivalent strength should be found as:

$$f_{\mathrm{eq,Rd}} = f_{\mathrm{u}} / \gamma_{\mathrm{M2}} \tag{7.6}$$

(3) Where Formulae (7.2) to (7.5) define the controlling conditions in the shell, the von Mises equivalent design strength should be taken instead:

$$f_{\mathrm{eq,Rd}} = f_{\mathrm{y}} / \gamma_{\mathrm{M0}} \tag{7.7}$$

(4) The partial factors for resistance γ_{M0} and γ_{M2} should be as defined in 4.4.

(5) Where no application standard exists for the form of construction involved, or the application standard does not define the relevant values of γ_{M0}, the value of γ_{M0} should be taken from Table 4.1.

(6) Where the material has a nonlinear stress-strain curve, the value of the characteristic yield strength f_{yk} should be taken as the 0,2% proof stress.

(7) The resistance of bolted or riveted plates should be evaluated according to the provisions of EN 1993-1-8.

(8) The resistance of welded lap joints should be evaluated using the joint efficiency factor j_i according to Figure 7.1 in which $t_1 \geq t_2$ with $\ell/t_1 \geq 5$. The joint efficiency factor j_i should be determined as follows:

— for a single-sided lap: $j_2 = 0{,}35$ under tension and $j_2 = 0$ under compression;

— for a double-sided lap: $j_1 = 1{,}0$ for $\ell/t_1 \geq 15$ and $j_1 = 1{,}25(1 - 3t_1/\ell)$ for $5 \leq \ell/t_1 < 15$.

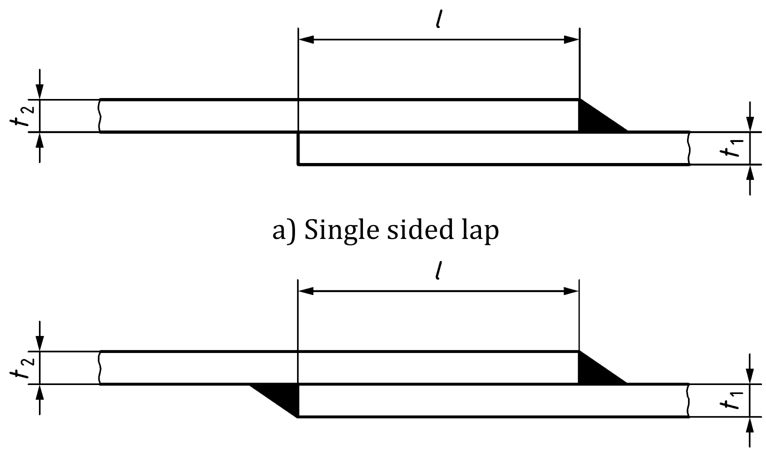


Figure 7.1 — Joint efficiency of lap joints (LS1)

(9) The resistance of the welded lap joint should be taken as:

$$\sigma_{x,Rd} \text{ or } \sigma_{\theta,Rd} = j_i f_y / \gamma_{M0} \tag{7.8}$$

(10) The effect of fastener holes in bolted or riveted construction should be taken into account in accordance with the provisions of EN 1993-1-1 and EN 1993-1-8 for both tension and compression. For the tension check, the resistance should be based on the design value of the ultimate strength f_u.

NOTE: For the resistance of a double sided lap joint under compression, see D.6.

7.2.3 Stress limitation

(1) In every verification of this limit state, the design stresses shall satisfy the condition:

$$\sigma_{eq,Ed} \le f_{eq,Rd} \text{ or } \sigma_{x,Ed} \le \sigma_{x,Rd} \text{ or } \sigma_{\theta,Ed} \le \sigma_{\theta,Rd} \tag{7.9}$$

as appropriate.

7.3 Design by computational MNA or GMNA analysis

(1) The design plastic failure resistance shall be determined as a load factor $R_{pl,d}$ applied to the design values F_{Ed} of the combination of actions for the relevant load case.

(2) The design values of the actions F_{Ed} should be determined as detailed in 7.1. The relevant load cases should be formed according to the required load combinations.

(3) In an MNA or GMNA analysis based on the design yield strength f_{yd}, the shell should be subject to the design values of the load cases detailed in (2), progressively increased by the load ratio R until the plastic failure limit condition is reached at R_{plf}.

(4) Where a GMNA analysis is used, if the analysis predicts a maximum load followed by a descending path, the maximum value should be used to determine the load ratio R_{GMNA}. Where a GMNA analysis does not predict a maximum load, but produces a progressively rising action-displacement relationship, the load ratio R_{GMNA} should be taken as no larger than the value at which the maximum von Mises equivalent plastic true strain in the structure attains the value

$$\varepsilon_{mps} = 0,04 - f_y / 40\,000 \tag{7.10}$$

where f_y is in MPa (N/mm^2).

NOTE The total maximum von Mises equivalent plastic true strain has been set at a value corresponding to approximately 50% of the lower bound to the strain at tensile rupture of a wide range of steels of different grades.

(5) The characteristic plastic failure resistance $R_{\mathrm{plf,k}}$ should be taken as either R_{MNA} or R_{GMNA} according to the analysis that has been used.

(6) The design plastic failure resistance $F_{R,plf,d}$ shall be obtained from:

$$F_{\mathrm{R,plf,d}} = \frac{F_{\mathrm{R,plf,k}}}{\gamma_{\mathrm{M0}}} = \frac{R_{\mathrm{plf,k}} \cdot F_{\mathrm{Ed}}}{\gamma_{\mathrm{M0}}} = R_{\mathrm{plf,d}} \cdot F_{\mathrm{Ed}} \tag{7.11}$$

where

γ_{M0} is the partial factor for resistance to plasticity as defined in 4.4.

(7) It shall be verified that:

$$F_{\mathrm{Ed}} \leq F_{\mathrm{R,plf,d}} = R_{\mathrm{plf,d}} \cdot F_{\mathrm{Ed}} \quad \text{or} \quad R_{\mathrm{plf,d}} \geq 1 \tag{7.12}$$

7.4 Design using standard formulae

(1) For each shell segment in the structure represented by a basic loading case as given by Annex A, the highest von Mises equivalent membrane stress $\sigma_{\mathrm{eq,Ed}}$ determined under the design values of the actions F_{Ed} should be limited to the stress resistance according to 7.2.3.

(2) For each shell or plate segment in the structure represented by a basic load case as given in Annex B, the design value of the actions F_{Ed} should not exceed the resistance F_{Rd} based on the design yield strength f_{yd}.

(3) Where net section failure at a bolted joint is a design criterion, the design value of the actions F_{Ed} should be determined for each joint. Where the stress can be represented by a basic load case as given in Annex A, and where the resulting stress state involves only membrane stresses, F_{Ed} should not exceed the resistance F_{Rd} based on the design ultimate strength f_{ud}, see 7.2.2(10).

8 Cyclic plasticity Limit State (LS2)

8.1 Design values of actions

(1) Unless an improved definition is used, the values of the actions for each load case should be chosen as the design values of those parts of the total actions that are expected to be applied and removed more than three times in the design life of the structure.

(2) Where an elastic analysis or the formulae from Annex C are used, only the varying part of the actions between the extreme upper and lower values should be taken into account.

(3) Where a materially nonlinear computer analysis is used, the varying part of the actions between the extreme upper and lower values should be considered to act in the presence of the design values of the coexistent permanent parts of the load.

8.2 Stress design

8.2.1 Design values of stress range

(1) The shell should be analysed using an LA or GNA analysis of the structure subject to the two extreme design values of the actions F_{Ed}. For each extreme load condition in the cyclic process, the stress components should be evaluated. From adjacent extremes in the cyclic process, the design values of the change in each stress component $\Delta\sigma_{\mathrm{x,Ed,i}}$, $\Delta\sigma_{\theta\mathrm{,Ed,i}}$, $\Delta\tau_{\mathrm{x\theta,Ed,i}}$ on each shell surface (represented as i=1,2 for the inner and outer surfaces of the shell) and at any point in the structure should be determined. From these changes in stress, the design value of the von Mises equivalent stress range on the inner and outer surfaces should be found from:

$$\Delta\sigma_{\mathrm{eq,Ed,i}} = \sqrt{\Delta\sigma_{\mathrm{x,Ed,i}}^{2} - \Delta\sigma_{\mathrm{x,Ed,i}} \cdot \Delta\sigma_{\theta\mathrm{,Ed,i}} + \Delta\sigma_{\theta\mathrm{,Ed}}^{2} + 3\Delta\tau_{\mathrm{x\,\theta,Ed,i}}^{2}} \tag{8.1}$$

(2) The design value of the stress range $\Delta\sigma_{eq,Ed}$ should be taken as the largest change in the von Mises equivalent stress changes $\Delta\sigma_{eq,Ed,i}$, considering each shell surface in turn (i = 1 and i = 2 considered separately).

(3) At a junction between shell segments, where the analysis models the intersection of the middle surfaces and ignores the finite size of the junction, the stress range may be taken at the first physical point inside the shell segment (as opposed to the value calculated at the intersection of the two middle surfaces).

NOTE This allowance is relevant where the stress changes very rapidly close to the junction and the connected plates can have a thickness that is significant relative to the stress gradient, which can be assessed as the stress peak divided by the stress gradient.

8.2.2 Design values of resistance

(1) The von Mises equivalent stress range resistance $\Delta f_{eq,Rd}$ should be determined from:

$$\Delta f_{eq,Rd} = 1{,}5 f_{yd} \tag{8.2}$$

in which

$$f_{yd} = f_y / \gamma_{M4} \tag{8.3}$$

(2) The partial factor for resistance γ_{M4} should be as defined in 4.4.

8.2.3 Stress range limitation

(1) In every verification of this limit state, the design stress range shall satisfy:

$$\Delta\sigma_{eq,Ed} \leq \Delta f_{eq,Rd} \tag{8.4}$$

8.3 Design by computational GMNA analysis

8.3.1 Design values of total accumulated plastic strain

(1) Where a geometrically and materially nonlinear computational analysis (GMNA) is used, the shell should be subject to the design values of the varying and permanent actions detailed in 8.1. A GMNA analysis should be used with an ideal elastic-plastic stress-strain curve.

(2) The total accumulated von Mises equivalent plastic strain $\varepsilon_{p,eq,Ed}$ at the end of the design life of the structure should be assessed.

(3) Plastic straining in any direction should always be treated as positive, so that plastic straining always leads to an increase in the total accumulated plastic strain.

(4) The total accumulated von Mises equivalent plastic strain may be determined using an analysis that models all cycles of loading during the design life.

(5) Unless a more refined analysis is carried out, the total accumulated von Mises equivalent plastic strain $\varepsilon_{p,eq,Ed}$ may be determined from:

$$\varepsilon_{p,eq,Ed} = N_{cp} \Delta\varepsilon_{p,eq,Ed} \tag{8.5}$$

where

N_{cp}	is the number of cycles of loading inducing yielding in the design life of the structure;
$\Delta\varepsilon_{p,eq,Ed}$	is the largest increment in the von Mises equivalent plastic strain during one complete load cycle at any point in the structure, occurring after the third cycle.

(6) It may be assumed that “at any point in the structure” means at any point not closer to a notch or local discontinuity than the thickness of the thickest adjacent plate.

8.3.2 Total accumulated plastic strain limitation

(1) Unless a more sophisticated low cycle fatigue assessment is undertaken, the design value of the total accumulated von Mises equivalent plastic strain $\varepsilon_{p,eq,Ed}$ should satisfy the condition:

$$\varepsilon_{p,eq,Ed} \le a_{p,eq} \left(0,04 - f_{yd} / 40\,000\right) \tag{8.6}$$

where f_{yd} is the design value of the yield stress according to 8.2.2.

The value of $a_{p,eq}$ should be taken as $a_{p,eq}$ = 2, unless relevant test data shows that a higher value is appropriate.

NOTE The partial factor on the yield stress has been reduced for cyclic plasticity γ_{M4}. The total acceptable von Mises equivalent plastic strain ε_{mps} given in 7.3 is here reduced by the factor $a_{p,eq}$ to take some account of the differences between cyclic and monotonic straining.

8.4 Design using standard formulae

(1) For each shell segment in the structure, represented by a basic loading case as given by Annex C, the highest von Mises equivalent stress range $\Delta\sigma_{eq,Ed}$ considering both shell surfaces under the design values of the actions F_{Ed} should be determined using the relevant formulae given in Annex C. The further assessment procedure should be as detailed in 8.2.

9 Buckling Limit State (LS3)

9.1 Design values of actions

(1) All relevant combinations of actions causing compressive membrane stresses or shear membrane stresses in the shell wall shall be taken into account.

9.2 Special definitions and symbols

(1) See the special definitions of terms concerning buckling in 3.1.7.

(2) In addition to the symbols defined in 3.2, additional symbols should be used in Clause 9 as set out in (3) and (4).

(3) The stress resultant and stress quantities should be taken as follows:

$n_{x,Ed}$, $\sigma_{x,Ed}$ are the design values of the acting buckling-relevant meridional (axial) membrane stress resultant and stress (positive when in compression);

$n_{\theta,Ed}$, $\sigma_{\theta,Ed}$ are the design values of the acting buckling-relevant circumferential (hoop) membrane stress resultant and stress (positive when in compression);

$n_{x\theta,Ed}$, $\tau_{x\theta,Ed}$ are the design values of the acting buckling-relevant shear membrane stress resultant and stress.

(4) Buckling resistance parameters for use in stress design:

$\sigma_{x,Rcr}$ is the meridional (axial in a cylinder) elastic critical buckling stress;

$\sigma_{\theta,Rcr}$ is the circumferential elastic critical buckling stress;

$\tau_{x\theta,Rcr}$ is the shear elastic critical buckling stress;

$\sigma_{x,Rk}$ is the meridional characteristic buckling stress;

$\sigma_{\theta,Rk}$ is the circumferential characteristic buckling stress;

$\tau_{x\theta,Rk}$ is the shear characteristic buckling stress;

$\sigma_{x,Rd}$ is the meridional (axial in a cylinder) design buckling stress;

$\sigma_{\theta,Rd}$ is the circumferential design buckling stress;

$\tau_{x\theta,Rd}$ is the shear design buckling stress.

NOTE This is a special convention for shell design that differs from that detailed in EN 1993-1-1.

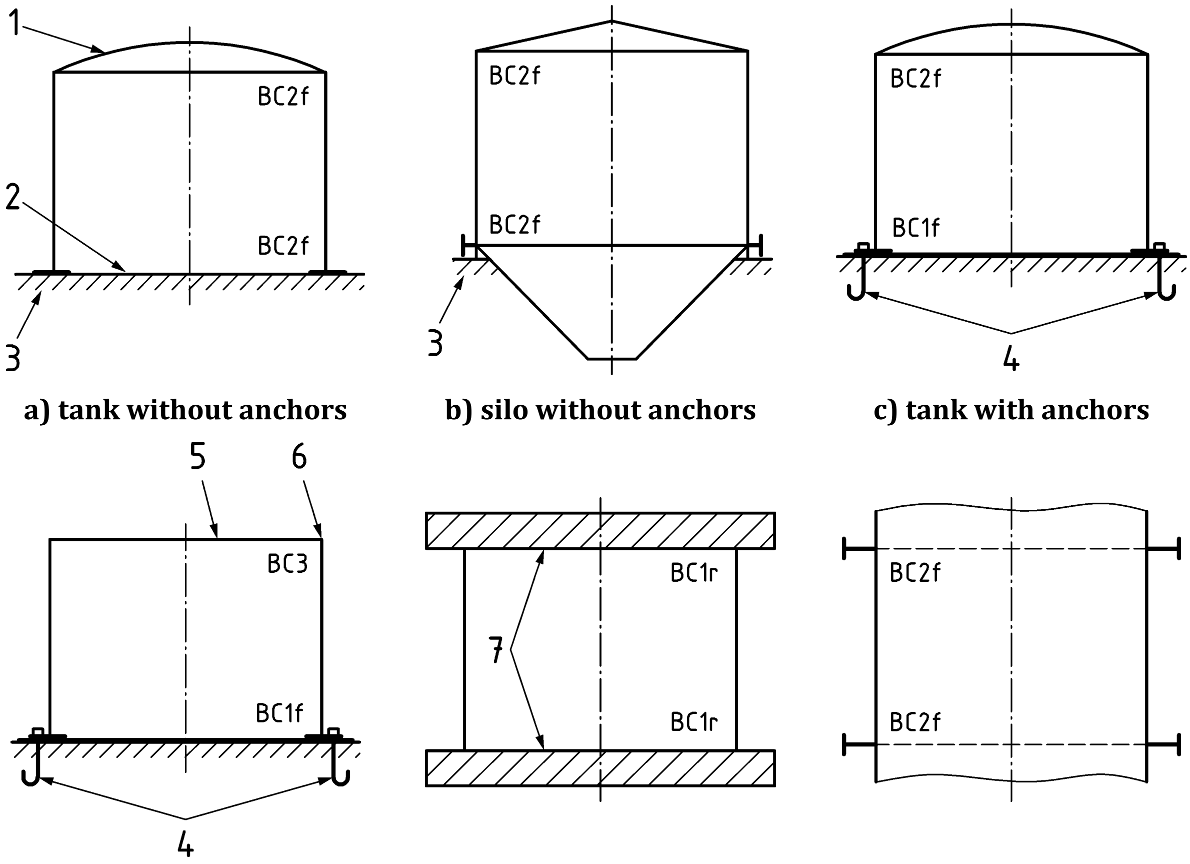
(5) The sign convention for use with LS3 should be taken as compression positive for meridional and circumferential stresses and stress resultants.

9.3 Buckling-relevant boundary conditions

(1) For the buckling limit state, special attention should be paid to the boundary conditions which are relevant to the incremental displacements during buckling (as opposed to pre-buckling displacements).

(2) Examples of situations in which the different simple boundary conditions of Table 6.1 arise are illustrated in Figures 9.1 and 9.2. These conditions apply only to the restraint of buckling displacements.

NOTE Displacements induced during the principal loading (pre-buckling displacements) place a weak demand on strict adherence to the precise boundary condition. Thus, full fixity is rarely achieved and is not critical to the pre-buckling condition where the chief design consideration is the magnitude of induced stresses. But the ultimate limit state of buckling in shells places a much greater burden on strict attainment of the assumed boundary conditions. This consideration is critical to a safe design to LS3.



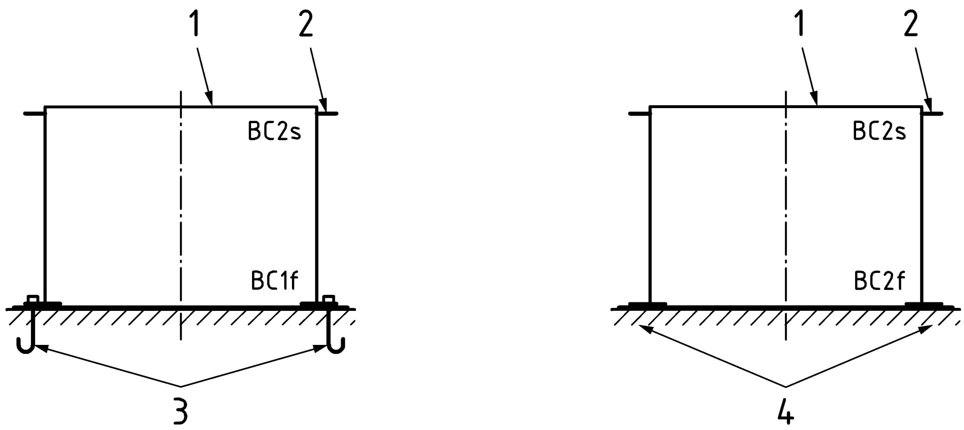
a) tank without anchors **b) silo without anchors** **c) tank with anchors**

d) open tank with anchors **e) laboratory experiment** **f) section of long ring-stiffened cylinder**

Key

1 roof
2 bottom plate
3 no anchoring
4 closely spaced anchors
5 open
6 no stiffening ring
7 end plates with high bending stiffness

Figure 9.1 — Schematic examples of simple boundary conditions for LS3



a) open tank with anchors and eaves stiffening ring **b) open tank without anchors and with eaves stiffening ring**

Key

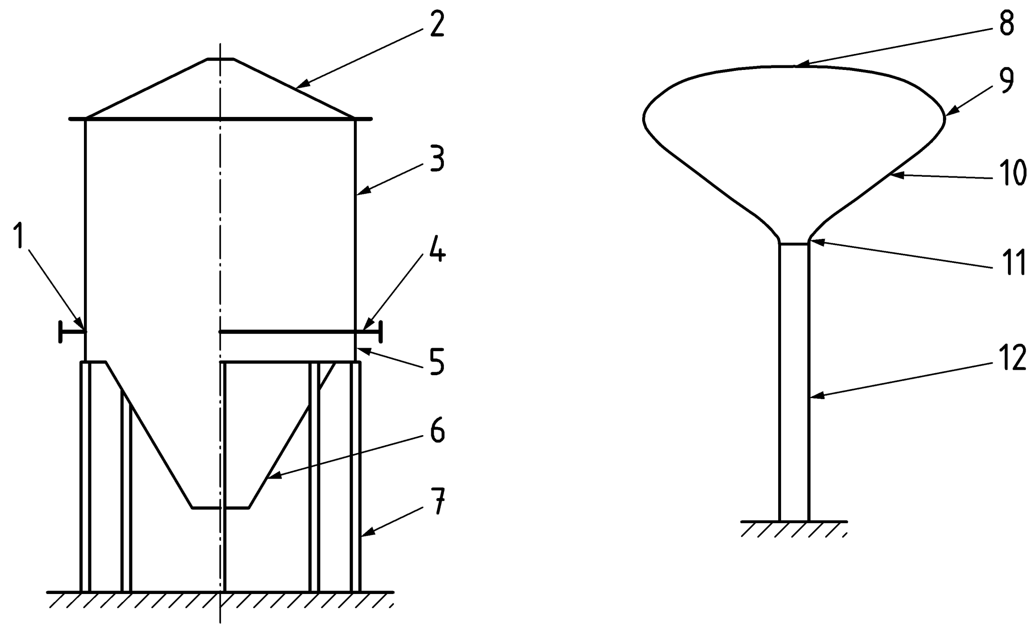
1 open

2 stiffening ring

3 closely spaced anchors

4 no anchors

Figure 9.2 — Schematic examples of ring boundary conditions for LS3



a) multi-segment elevated silo b) multi-segment pedestal tank

Key

1	Transition	7	Column
2	Conical roof	8	Spherical dome
3	Cylindrical shell	9	Toroid
4	Ring	10	Cone
5	Skirt	11	Toroid
6	Conical hopper	12	Cylinder

Figure 9.3 — Schematic examples of multi-segment shells

9.4 Buckling-relevant geometrical tolerances

9.4.1 General

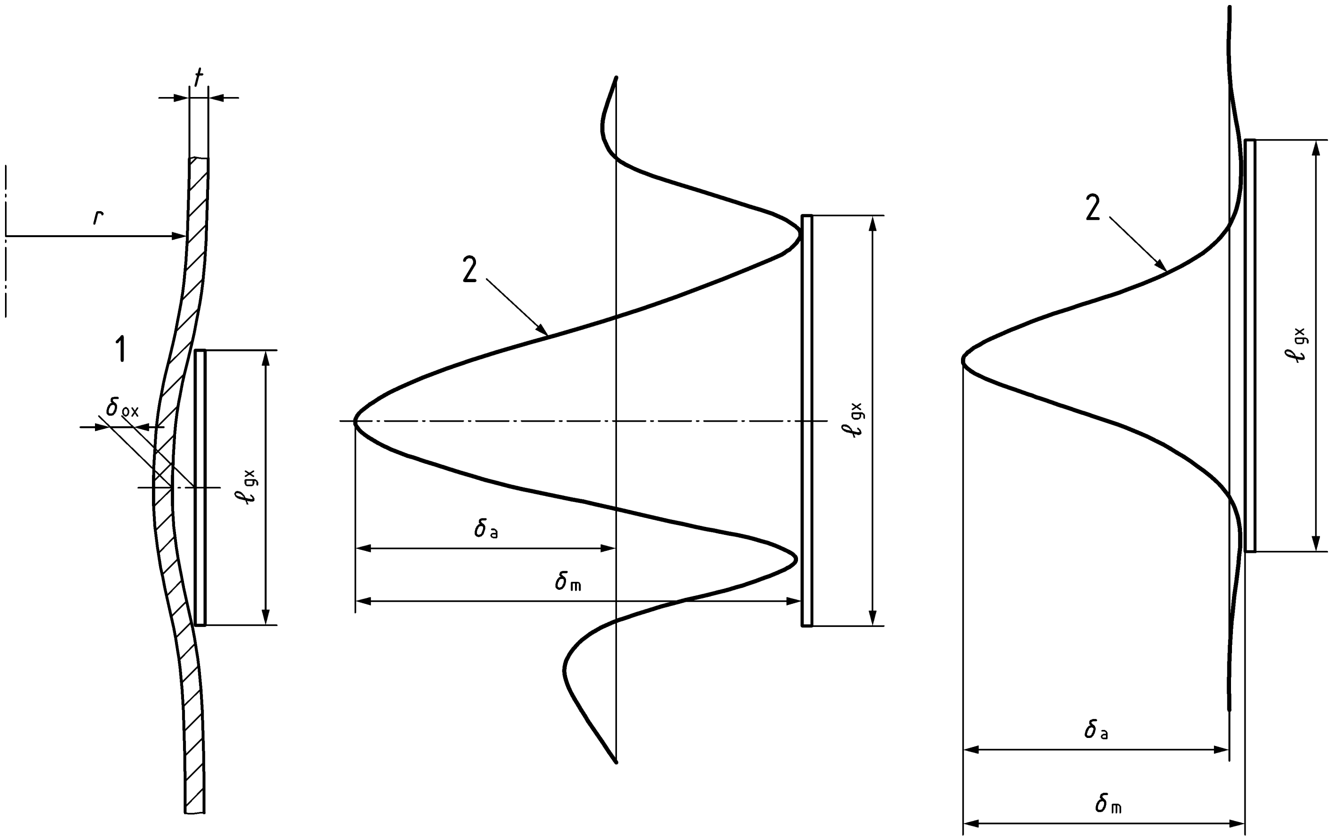
(1) Further to the erection tolerances defined in EN 1990-2, the following tolerance requirements for specific stress conditions in the shell should be taken into account.

(2) Unless further specific buckling-relevant geometrical tolerances are given in the relevant application parts of EN 1993, the following tolerance limits should be considered where design for LS3 is a requirement for the structure. These tolerance limits relate to specific uniform stress states that can be the primary cause of buckling in the shell. Some tolerance limits are therefore potentially not relevant to structures in which different stress states are found.

(3) Where GMNIA analyses are performed using assumed equivalent imperfections (see 9.8.2), the amplitudes of the imperfections should be chosen in a manner that is consistent with the tolerance measurements defined in this sub-clause 9.4.1.

NOTE 1 A consistent choice of imperfection amplitude means that, when the assumed imperfection form is presented with an imperfection measurement as defined in this sub-clause 9.4.1, it is required to achieve the defined chosen amplitude according to 9.8.2 (33). It is not appropriate to simply multiply an eigenmode imperfection (characterised by a unit amplitude) by the defined assumed imperfection amplitude 9.8.2 (33) as this usually results in deeper imperfections than that obtained using the tolerance measurement (typically around double the required value).

NOTE 2 In Figures 9.4b) and c), the measurement δ_a corresponds to the algebraic amplitude of an imperfection defined by a shape derived from a computational analysis. The measurement δ_m in Figures 9.4b) and c) corresponds to the measured amplitude using the tolerance measurement of Figure 9.4a). It is important that the defined amplitude used in computational assessments corresponds to the tolerance measurement δ_m.



$\delta_m \approx 2\delta_a$ $\delta_m \approx \delta_a$

a) Tolerance measurement b) Typical eigenmode c) Weld depression

Key

1 inward

2 Shell geometry

Figure 9.4 — Consistent choice of imperfection amplitude, using ℓ_{gx} as an example

(4) The tolerance requirements for LS3 are divided into four categories, as set out in 9.4.3 to 9.4.6. Not all of these tolerance requirements are appropriate to all shells, as the needs depend on the stress states that will develop within the shell.

(5) Each tolerance is therefore identified below for its relevance to a particular structure. The tolerances that are required to meet the resistance requirements should be clearly communicated to the fabricator and the relevant authority as part of the design documentation.

NOTE 1 The geometric tolerances given here are those that are known to have a large impact on the resistance of the structure under specific loading conditions.

NOTE 2 The characteristic buckling resistances determined hereafter are based on imperfection forms and amplitudes that relate to geometric tolerances that are expected to be met during the execution of welded structures. Other construction forms are expected to have similar or greater resistances.

(6) The fabrication tolerance quality class should be chosen as Class A, Class B or Class C according to the tolerance definitions in 9.4.3 to 9.4.6. The description of each class relates only to the resistance evaluation and not to other considerations (e.g. aesthetics or functional performance).

NOTE 1 The tolerances defined here currently correspond closely to those specified in the execution standard EN 1090-2, but not all are needed for every structure. These tolerance requirements relate to specific failure modes, so distinctions are required to be considered between the categories and amplitudes according to the susceptibility of the structure to each potential failure mode in relation to its geometry, loading condition and boundary conditions. The requirements are consequently more fully defined in this standard than in EN 1090-2.

NOTE 2 The relationship between each imperfection amplitude and the assessed resistance is vital to verifying the resistance, so it can be acceptable to use a lower fabrication quality for some categories of imperfection, whilst still maintaining the required resistance for a higher fabrication quality class.

NOTE 3 The tolerances are also clearly defined here to permit the designer to exploit opportunities for high control of fabrication processes (e.g. in factory environments) to achieve more economic designs.

NOTE 4 The tolerances defined here are also required for the application of defined imperfection assumptions in GMNIA analyses given in 9.8. For application in 9.8, the adopted imperfection amplitude is related to the manner in which the tolerance is specified in this sub-clause 9.4.

(7) Each imperfection category should be classified separately: the quality class of the complete structure should be defined as the lowest tolerance category required amongst all the buckling modes found to be relevant to the structural integrity. Tolerances that relate to buckling modes that are far from being critical to the structural resistance may be set at a lower quality class without affecting the assessed quality class of the complete structure.

(8) The different tolerance categories may each be treated independently, and no interactions need normally be considered.

(9) The stress state cases for which each tolerance category is required is shown in Table 9.1.

Table 9.1 — Required tolerance dependent on the shell stress state

Stress state dominated by membrane stresses	Out of roundness	Unintended eccentricity	Dimple	Interface flatness
Meridional (axial) compression	Applies	Applies	l_{gx} and l_{gw} only	Applies
Circumferential compression	Applies		$l_{g\theta}$ and l_{gw} only	
Shear	Applies			

NOTE It is good practice to make all tolerances meet at least those of Fabrication Quality Class C, though some of these can be for aesthetic rather than structural integrity reasons.

(10) It should be established by representative sample checks on the completed structure that the measurements of the geometrical imperfections are within the geometrical tolerances required for the structure by the designer according to the methods and values stipulated in 9.4.3 to 9.4.6.

(11) A sample may be regarded as representative if the tolerance is verified at all critical locations.

(12) The design documentation should specify and identify the critical locations in the structure at which particular tolerance measurements are most critical to the structural integrity. This requirement is made to avoid the use of excessive tolerance measurement on a completed structure.

(13) All sample imperfection measurements should be undertaken on the unloaded structure (except for self-weight) and, where possible, with the operational boundary conditions.

NOTE Where appropriate, misalignment and welding dimples can be measured in the factory and are deemed to be of sufficient accuracy to reliably represent those in the completed erected structure.

(14) Where the tolerance measurement systems indicated in Figures 9.5 to 9.8 are used, the strict interpretations given in 9.4.3 to 9.4.6 are relevant.

(15) Where a laser scan of the completed unloaded structure has been undertaken, the results can be interpreted numerically for each tolerance by using a notional measuring gauge of the defined length according to 9.4.3 to 9.4.6. Where this evaluation method is used, a best fit log normal distribution of the amplitudes of measurements relating to each tolerance type should be used to identify the 95%ile value. It is this percentile value that should be assessed against the relevant defined tolerance limit.

(16) If the measurements of geometrical imperfections do not satisfy the geometrical tolerances stated in 9.4.3 to 9.4.6, the specific location or locations where this is found should be examined. The obtained measurements should then be used to assess the buckling resistance at each location using the strict procedures of this Clause 9. The assessed buckling resistance may then be compared with the design requirement for resistance at that location to determine whether the failure to meet the geometrical tolerance requirement presents a risk to the structural integrity, using the rules of this Clause 9 and the resistance definitions of Annexes D and E.

(17) Where it can be shown by introducing the measured imperfections into the design resistance formulae (see 9.5 to 9.7 and Annexes D and E), or by using GMNIA calculations (see 9.8), that no risk exists, the strict demand with respect to the specific tolerance requirement may be relaxed, as specified by the relevant authority or, where not specified, as agreed for the specific project by the relevant parties.

NOTE This provision is made in consideration of the character of stepped wall construction widely used for shell structures. It is a common situation that the thinner plate above a change of plate thickness has a greater resistance than is strictly necessary to meet the buckling resistance requirement. The thicker plate below the change has a considerably increased resistance, so the achieved resistance of the thinner plate, albeit with deeper imperfections than meet the tolerance requirement, can still be adequate to achieve the required structural integrity. Where the procedures of (16) and (17) indicate that an adequate resistance exists throughout the structure, correction steps are unnecessary. Such correction steps can even be detrimental to the structural resistance due to the potential for increased imperfections and residual stresses.

(18) If the measurements of geometrical imperfections do not satisfy the geometrical tolerances stated in 9.4.3 to 9.4.6, and correction steps are deemed necessary, procedures such as straightening or the addition of stiffeners should be investigated and decided individually.

(19) Before a decision is made in favour of straightening to reduce geometric imperfections, the potential should be considered for the remedial measures to induce additional residual stresses and additional distortions in the shell.

9.4.2 Assessment of the dominant membrane stress at any location

(1) The following procedure may be used to limit the number of tolerance checks that are required on the completed structure.

(2) For any load case applied to a shell that is subject to assessment against LS3 (buckling), the locations at which the design values of the acting membrane stresses most closely approach the corresponding buckling resistance should be identified.

(3) The 2D membrane stress state at the location being evaluated should be examined for its different components: meridional (axial) compression $\sigma_{x,Ed}$, circumferential compression $\sigma_{\theta,Ed}$ and membrane shear $\tau_{x\theta,Ed}$. The corresponding design buckling resistances at the same location should be found, using the relationships of Annexes D and E, as $\sigma_{x,Rd}$, $\sigma_{\theta,Rd}$, and $\tau_{x\theta,Rd}$. Interactions between compression and shear in these design buckling resistances may be ignored. The exclusion zones defined in D.4.3 may also be excluded.

(4) The ratio of each acting membrane stress to its resistance should be found as:

$$k_x = \sigma_{x,Ed} / \sigma_{x,Rd} \tag{9.1}$$

$$k_\theta = \sigma_{\theta,Ed} / \sigma_{\theta,Rd} \tag{9.2}$$

$$k_{x\theta} = \tau_{x\theta,Ed} / \tau_{x\theta,Rd} \tag{9.3}$$

(5) The largest value amongst k_x, k_θ and $k_{x\theta}$, should be identified and termed k_{max}. The ratios of the two other k_i values to k_{max} should then be found as $s_{irat} = k_i/k_{max}$.

(6) Where neither of the two values of s_{irat} exceeds 0,30, the stress state may be deemed to be dominated by a single compressive membrane stress resultant, termed the dominant membrane stress resultant, and the limited tolerance demands identified in Table 9.1 for that membrane stress resultant may be used.

(7) Where one of the ratios s_{irat} exceeds 0,30, the tolerance demands identified in Table 9.1 for both the k_{max} and the second identified membrane stress resultant should be applied.

(8) Where (6) or (7) do not identify a dominant compressive membrane stress resultant, all tolerance requirements at that location should be met.

NOTE A single dimple, even at the critical location, is not sufficient to reduce the buckling resistance down to the value defined as the characteristic value defined in Annex D. Even under uniform axial compression, a dimple of the critical amplitude must extend over a significant part of the circumference to reduce the buckling load to the defined value.

9.4.3 Out-of-roundness tolerance

(1) The out-of-roundness tolerance is important where the identified most critical mode in the shell arises under circumferential compression. Where other membrane stresses are dominant according to 9.4.2, this tolerance measurement may be classed as belonging to one fabrication tolerance class higher than the following provisions indicate.

(2) The out-of-roundness should be assessed in terms of the parameter U_r (see Figure 9.5) given by:

$$U_r = \frac{d_{max} - d_{min}}{d_{nom}} \tag{9.4}$$

where

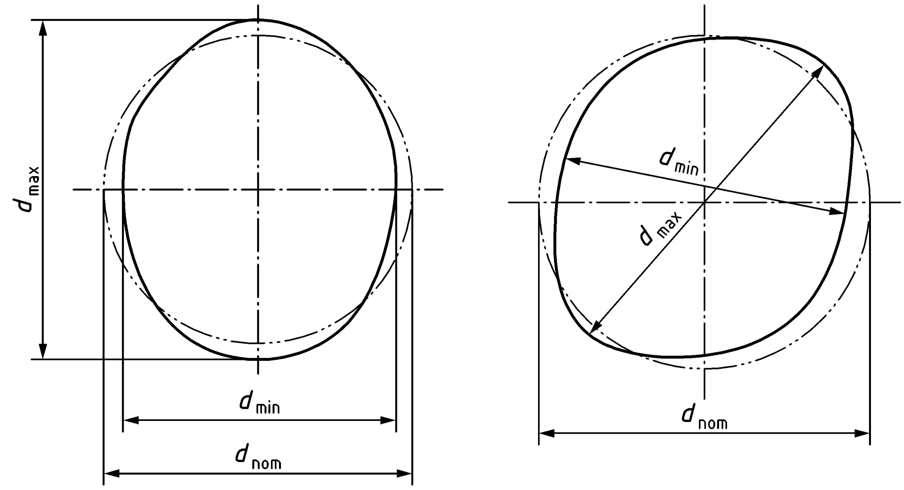
d_{max} is the maximum measured diameter;

d_{min} is the minimum measured diameter;

d_{nom} is the nominal diameter.

NOTE These diameters can be measured either on the inside or on the outside, as appropriate to the structure.

(3) The measured internal diameter from a given point should be taken as the largest distance across the shell from the point to any other internal point at the same axial coordinate sufficient number of diameters should be measured to identify the maximum and minimum values.



a) flattening or elliptical b) indeterminate

Figure 9.5 — Measurement of diameters for assessment of out-of-roundness

NOTE This tolerance is principally concerned with limitation of the highest local radius of curvature. which affects all buckling resistances strongly. Well-chosen sample measurements can give clear information on this parameter.

(4) The out-of-roundness parameter U_r should satisfy the condition:

$$U_r \leq U_{r,\max} \tag{9.5}$$

where

$U_{r,max}$ is the out-of-roundness tolerance parameter for the relevant fabrication tolerance quality class.

The values of $U_{r,max}$ are given in Table 9.2.

Table 9.2 — Values for out-of-roundness tolerance parameter $U_{r,max}$

	Diameter range	***d* [m] ≤ 0,50m**	**0,50m < *d* [m] < 1,25m**	**1,25m ≤ *d* [m]**
Fabrication tolerance quality class	**Description**	Value of $U_{r,max}$		
Class A	Excellent	0,014	0,007 + 0,0093(1,25−*d*)	0,007
Class B	High	0,020	0,010 + 0,0133(1,25−*d*)	0,010
Class C	Normal	0,030	0,015 + 0,0200(1,25−*d*)	0,015

9.4.4 Unintended eccentricity tolerance

(1) The unintended eccentricity tolerance is important in circumferential joints where the identified most critical mode in the shell arises under meridional (axial) compression. This tolerance may be reduced by one fabrication tolerance class where other stress states dominate the critical mode.

(2) Where bolted or riveted construction is used with lap joints, the tolerance defined here may be ignored provided the provisions of D.6 are included in the resistance evaluation.

(3) The unintended eccentricity tolerance in circumferential joints may be increased to designer-defined quantities where formal provision for joint eccentricity is made according to the provisions of D.5.1.2 and D.6.

(4) At joints in shell walls perpendicular to the dominant compressive membrane stress resultant, the unintended eccentricity should be evaluated from the measurable total eccentricity e_{tot} and the intended offset e_{int} from:

$$e_a = e_{tot} - e_{\mathrm{int}} \tag{9.6}$$

where

e_a is the unintended eccentricity between the middle surfaces of the joined plates, see Figure 9.6 a);

e_{int} is the intended offset between the middle surfaces of the joined plates, see Figure 9.6 b);

e_{tot} is the eccentricity between the middle surfaces of the joined plates, see Figure 9.6 c).

NOTE This tolerance principally affects the buckling resistance under axial compression or global bending, because the middle surfaces of the shell in the upper and lower parts are misaligned.

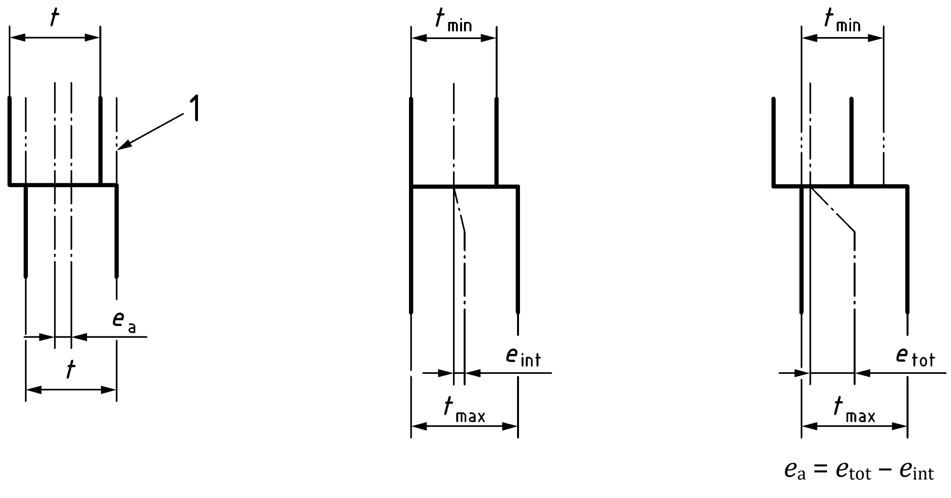
(5) Where this misalignment tolerance is found to be the critical controlling tolerance, the shell buckling resistance may alternatively be assessed using the rules on misalignment in Annex D, D.5 or on lap joints in Annex D, D.6.

(6) The unintended eccentricity e_a should also be assessed in terms of the unintended eccentricity parameter U_e given by:

$$U_{\mathrm{e}} = \frac{e_{\mathrm{a}}}{t_{\mathrm{av}}} \text{ or } U_{\mathrm{e}} = \frac{e_{\mathrm{a}}}{t} \tag{9.7}$$

where

t_{av} is the mean thickness of the thinner and thicker plates at the joint.



a) unintended eccentricity when there is no change of plate thickness

b) intended offset at a change of plate thickness without unintended eccentricity

c) total eccentricity (unintended plus intended) at change of plate thickness

Key

1 perfect joint geometry

Figure 9.6 — Unintended eccentricity and intended offset at a joint

(7) The unintended eccentricity parameter U_e should satisfy the condition:

$$U_e \leq U_{e,\max} \tag{9.8}$$

where

$U_{e,max}$ is the unintended eccentricity tolerance parameter for the relevant fabrication tolerance quality class.

The values of $U_{e,max}$ are given in Table 9.3.

Table 9.3 — Values for unintended eccentricity tolerances

Fabrication tolerance quality class	Description	Value of $U_{e,max}$
Class A	Excellent	0,14
Class B	High	0,20
Class C	Normal	0,30

NOTE Intended offsets are treated within D.5.1.2 and lapped joints are treated within D.6. These two cases are not treated as imperfections within this standard.

9.4.5 Dimple tolerances

(1) The dimple tolerance is important for compressive membrane stresses in all directions, but the tolerance measurement relevant to each direction of stress is different. The tolerance requirement for a specific measurement may be reduced by one fabrication tolerance class when the procedure of 9.4.2 shows that the specific measurement relates to a membrane stress state that does not dominate the critical mode.

(2) A dimple measurement gauge should be used in every position in both the meridional (axial) (Figure 9.7) and circumferential (Figure 9.8) directions. The meridional gauge should be straight, but the gauge for measurements in the circumferential direction should have a curvature equal to the intended radius of curvature r of the middle surface of the shell.

(3) Where bolted or riveted construction is used with lap joints, the tolerance defined here may be reduced to 50% of the defined values, provided that the provisions of D.6 are included in the resistance evaluation.

(4) The depth δ_0 of initial dimples in the shell wall should be measured using gauges of length ℓ_g which should be taken as follows:

a) Wherever meridional (axial) compressive membrane stresses have been found to be significant in the assessment of the resistance of the structure according to 9.4.1 and 9.4.2, measurements should be made in both the meridional and circumferential directions, including across welds, using the gauge of length ℓ_{gx} given by:

$$\ell_{gx} = 4\sqrt{rt} \tag{9.9}$$

b) Where the shell radius to thickness ratio is less than r/t = 400, measurements in the meridional direction should be made across circumferential welds, using the gauge length ℓ_{gw}:

$$\ell_{gw} = 25t \quad \text{or} \quad \ell_{gw} = 25t_{min}\text{, but with } \ell_{gw} \leq 500 \text{ mm} \tag{9.10}$$

where

$t_{\min}$ is the thickness of the thinner plate at the weld.

c) Where circumferential compressive membrane stresses or in-plane shear stresses have been found to be significant in the assessment of the resistance of the structure according to 9.4.1 and 9.4.2, circumferential direction measurements should be made using the gauge of length $\ell_{g\theta}$ given by:

$$\ell_{g\theta} = 2,3\left(\ell_S^2\ rt\right)^{0,25}\text{, but } \ell_{g\theta} \le r \tag{9.11}$$

where

ℓ_S is the meridional length of the shell segment between boundaries that are either BC1 or BC2.

(5) Where the shell has substantial intermediate ring stiffeners to keep the shell circular, the ring may be taken to provide a BC1f boundary condition.

NOTE The shell above and below an intermediate ring stiffener provide axial restraint to a buckle, so the boundary condition can be BC1f rather than BC2f.

(6) Except as specified in Annex D, a ring stiffener may be considered to be substantial if it causes the critical buckle in an LBA analysis to be contained within the shell without noticeable deformation of the stiffener from its circular shape.

NOTE Some guidance on the required stiffness can be obtained from D.5.3.4 and other information from EN 1993-4-1.

(7) The depth of initial dimples should be assessed in terms of the dimple parameters U_{0x}, $U_{0\theta}$, U_{0w} given by:

$$U_{0x} = \delta_{0x} / \ell_{gx} \tag{9.12}$$

$$U_{0w} = \delta_{0w} / \ell_{gw} \tag{9.13}$$

$$U_{0\theta} = \delta_{0\theta} / \ell_{g\theta} \tag{9.14}$$

(8) The value of the dimple parameters U_{0x}, U_{0w}, $U_{0\theta}$ should satisfy the conditions:

$$U_{0x} \le U_{0x,\max} \tag{9.15}$$

$$U_{0w} \le U_{0x,\max} \tag{9.16}$$

$$U_{0\theta} \le U_{0\theta,ref}\left(\frac{L}{r}\right)^{1/4}\left(\frac{t}{r}\right)^{3/8} \tag{9.17}$$

where

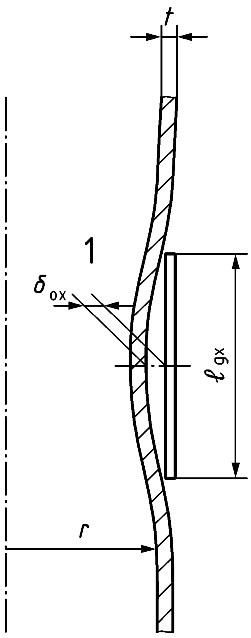
$U_{0x,\max}$ is the dimple tolerance parameter for axial compression for the relevant fabrication tolerance quality class.

$U_{0\theta,\mathrm{ref}}$ is the dimple tolerance reference value for circumferential compression for the relevant fabrication tolerance quality class.

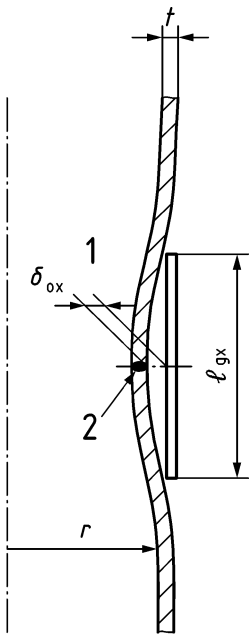
The values of $U_{0x,\max}$ and $U_{0\theta,\mathrm{ref}}$ are given in Table 9.4.

Table 9.4 — Values for dimple tolerance parameters $U_{0x,max}$ and $U_{0\theta,ref}$

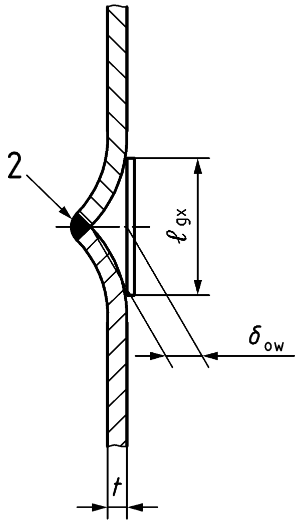
Fabrication tolerance quality class	Description	Value of $U_{0x,max}$	Value of $U_{0\theta,ref}$
Class A	Excellent	0,006	0,008
Class B	High	0,010	0,017
Class C	Normal	0,016	0,036



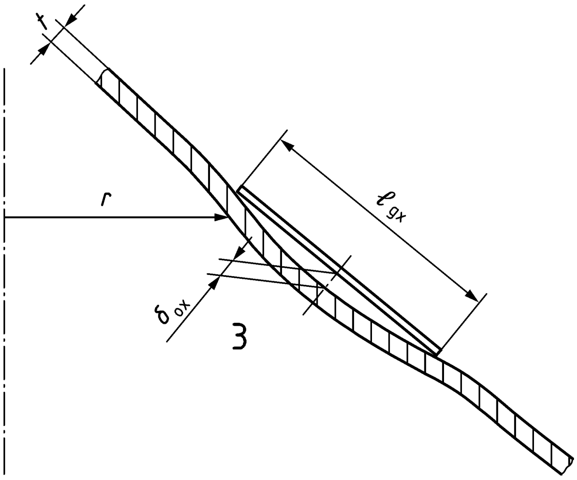
a) Measurement on a meridian (see 9.4.5(4) a)



b) First measurement on a meridian across a weld (see 9.4.5(4) a)



c) Second measurement across a weld with special gauge (see 9.4.5(4) b)



d) Measurement on a conical meridian (see 9.4.5(4) a)

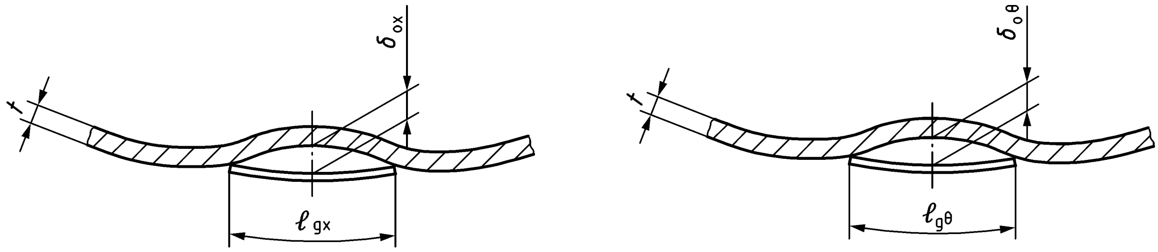
Key

1 Inward

2 Weld

3 Inward normal to the surface

Figure 9.7 — Measurement of depths δ_0 of initial dimples on the meridian



a) First measurement on a circumferential circle (see 9.4.5(4) a)

b) Second measurement on circumferential circle (see 9.4.5(4) c)

Figure 9.8 — Measurement of depths δ_0 of initial dimples on the circumference

9.4.6 Interface flatness tolerance

(1) The interface flatness tolerance is only important for axial compressive membrane stresses at the base of a shell. The tolerance requirement may be ignored when axial compression does not play a significant role in the assessed critical mode of buckling.

NOTE The potential for buckling under axial compression due to poor interface flatness is not limited to the zone immediately above the location where this tolerance is not met. The resulting deviation in compressive stresses can penetrate far up the shell.

(2) Where another structure continuously supports a shell, its deviation from flatness at the interface should be evaluated using measurements of the separation of the shell from its supporting structure.

(3) The separation of the shell from the foundation or supporting structure at any location should measured as δ_{u0} using a measuring gauge length of ℓ_g in the circumferential direction, in the style of Figure 9.8 a.

(4) The gauge length ℓ_g should be $\ell_g = 4\sqrt{rt}$ where r and t are the radius and wall thickness of the lowest course of the cylinder.

(5) The maximum allowable value of δ_{u0} is defined as $\delta_{u0,\max}$ given by:

$$\delta_{u0,\max} = 0,02\sqrt{rt} \quad \text{but} \leq 15 \text{ mm} \tag{9.18}$$

(6) This tolerance measurement is only relevant to limit the deviations of the foundation of a shell structure that is susceptible to buckling under axial compression.

9.5 Stress design

9.5.1 Design values of stresses

(1) The rules given here apply to a cylindrical shell segment that can be part of a larger structure. The shell length ℓ is taken as the length of the segment alone.

(2) The design values of stresses for the shell segment $\sigma_{x,Ed}$, $\sigma_{\theta,Ed}$ and $\tau_{x\theta,Ed}$ should be taken as the key values of these three basic compressive and shear membrane stresses obtained from a linear shell analysis (LA). Under purely axisymmetric conditions of loading and support, and in other simple load cases, membrane theory may be used.

(3) The key values of membrane stresses should be taken as the maximum value of each stress component at a single axial coordinate in the shell segment, unless specific provisions are given in Annex D or in the relevant application part of EN 1993.

NOTE In some cases where a membrane theory analysis has been used, the key values of membrane stresses are slightly larger than the real maximum values (e.g. stepped walls under circumferential compression, see D.5.3).

(4) For basic loading cases the three membrane stress components in the shell segment may be taken from Annex A or Annex C.

9.5.2 Design resistance (buckling strength)

(1) The elastic critical buckling stress for each component of membrane stress in the shell segment should be obtained as $\sigma_{x,Rcr}$, $\sigma_{\theta,Rcr}$ and $\tau_{x\theta,Rcr}$ from the relevant formulae in Annex D.

NOTE 1 Where the membrane stress state effectively only involves one of these three basic components, it is sufficient to address that alone (see 9.4.2), without conducting the following triple calculation for possible interactions between the different components.

NOTE 2 Where an LBA analysis is used in place of the relevant formulae in Annex D, the LBA will give a scale factor on the stress state which can be exploited to obtain the required critical values.

(2) Where no appropriate formulae are given in Annex D, the elastic critical buckling condition may be extracted from a computational LBA analysis of the shell segment under the buckling-relevant combinations of actions defined in 9.1 (1). For the conditions that this analysis should satisfy, see 9.7.2.2.

NOTE Where an LBA analysis is used, and no single stress component is clearly dominant in the critical state, it is sometimes more satisfactory to follow the LBA-MNA design process of 9.7.

(3) The relative slenderness of the shell segment $\overline{\lambda}$ for each stress component should be separately determined from:

$$\overline{\lambda} = \sqrt{f_{yk} / \sigma_{x,Rcr}} \tag{9.19}$$

$$\overline{\lambda}_{\theta} = \sqrt{f_{yk} / \sigma_{\theta,Rcr}} \tag{9.20}$$

$$\overline{\lambda}_{\tau} = \sqrt{(f_{yk} / \sqrt{3}) / \tau_{x\theta,Rcr}} \tag{9.21}$$

(4) The elastic-plastic buckling reduction factor for each component χ_x, χ_θ and χ_τ should be separately determined as a function of the corresponding relative slenderness of the shell segment $\overline{\lambda}$ = $\overline{\lambda}_x$, $\overline{\lambda}_\theta$, or $\overline{\lambda}_\tau$ from:

$$\chi = \chi_h - \left(\frac{\overline{\lambda}}{\overline{\lambda}_0}\right)(\chi_h - 1) \quad \text{when } \overline{\lambda} \le \overline{\lambda}_0 \tag{9.22}$$

$$\chi = 1 - \beta\left(\frac{\overline{\lambda} - \overline{\lambda}_0}{\overline{\lambda}_p - \overline{\lambda}_0}\right)^{\eta} \quad \text{when } \overline{\lambda}_0 < \overline{\lambda} < \overline{\lambda}_\mathrm{p} \tag{9.23}$$

$$\chi = \frac{\alpha}{\overline{\lambda}^2} \quad \text{when } \overline{\lambda}_\mathrm{p} \le \overline{\lambda} \tag{9.24}$$

where the relevant capacity parameters for each separate component of the membrane stress are:

α the elastic imperfection reduction factor for that component;

β the plastic range factor for that component;

η the interaction exponent for that component;

$\bar{\lambda}_0$ the squash limit relative slenderness for that component;

χ_h the hardening limit for that component.

(5) The value of the plastic limit relative slenderness $\bar{\lambda}_p$ for each component $\bar{\lambda}_{px}$, $\bar{\lambda}_{p\theta}$ and $\bar{\lambda}_{p\tau}$ should be determined, using the relevant values of α and β from:

$$\bar{\lambda}_p = \sqrt{\frac{\alpha}{1-\beta}} \tag{9.25}$$

(6) The value of η is sometimes defined as a single value, but in more general cases it is defined by two limiting values η_0 and η_p, with η determined as:

$$\eta = \left[\frac{\bar{\lambda}\left(\eta_p - \eta_o\right) + \bar{\lambda}_p \eta_o - \bar{\lambda}_o \eta_p}{\bar{\lambda}_p - \bar{\lambda}_o}\right] \tag{9.26}$$

where

η_0 is the value of η at $\bar{\lambda} = \bar{\lambda}_0$ for that component;

η_p is the value of η at $\bar{\lambda} = \bar{\lambda}_p$ for that component.

NOTE 1 The values of these capacity parameters for each component are given in Annex D.

NOTE 2 Formula (9.24) describes the elastic buckling stress, accounting for geometric imperfections. In cases where the behaviour is entirely elastic, the characteristic buckling stresses are directly defined by $\sigma_{x,Rk} = \alpha_x \sigma_{x,Rcr}$, $\sigma_{\theta,Rk} = \alpha_\theta \sigma_{\theta,Rcr}$, and $\tau_{x\theta,Rk} = \alpha_\tau \tau_{x\theta,Rcr}$.

(7) The characteristic buckling stress for each component should be obtained by multiplying the characteristic yield strength by the corresponding elastic-plastic buckling reduction factors χ:

$$\sigma_{x,Rk} = \chi_x f_{yk} \tag{9.27}$$

$$\sigma_{\theta,Rk} = \chi_\theta f_{yk} \tag{9.28}$$

$$\tau_{x\theta,Rk} = \chi_\tau f_{yk} \tag{9.29}$$

(8) The buckling resistance should be derived from the values for the three basic membrane stress components defined in 3.2.7 and Figure 3.3. The design buckling stress for each component should be obtained from:

$$\sigma_{x,Rd} = \sigma_{x,Rk} / \gamma_{M1} \tag{9.30}$$

$$\sigma_{\theta,Rd} = \sigma_{\theta,Rk} / \gamma_{M1} \tag{9.31}$$

$$\tau_{x\theta,Rd} = \tau_{x\theta,Rk} / \gamma_{M1} \tag{9.32}$$

(9) The partial factor for resistance of shell to stability γ_{M1} should be as defined in 4.4.

9.5.3 Stress limitation (buckling strength verification)

(1) Although buckling is not a purely stress-initiated failure phenomenon, the buckling limit state, within this sub-clause, should be represented by limiting the design values of compressive membrane stresses. The influence of bending effects on the buckling strength may be neglected provided they arise as a result of meeting boundary compatibility requirements. In the case of bending stresses from local loads or from thermal gradients, special consideration should be given.

(2) Depending on the loading and stress situation, one or more of the following checks for the key values of single membrane stress components should be carried out:

$$\sigma_{x,Ed} \leq \sigma_{x,Rd} \tag{9.33}$$

$$\sigma_{\theta,Ed} \leq \sigma_{\theta,Rd} \tag{9.34}$$

$$\tau_{x\theta,Ed} \leq \tau_{x\theta,Rd} \tag{9.35}$$

(3) If more than one of the three buckling-relevant membrane stress components are present under the actions under consideration, the following interaction check for the combined membrane stress state at any single location in the shell segment should be carried out:

$$\left(\frac{\sigma_{x,Ed}}{\sigma_{x,Rd}}\right)^{k_{ix}} - a_i\left(\frac{\sigma_{x,Ed}}{\sigma_{x,Rd}}\right)\left(\frac{\sigma_{\theta,Ed}}{\sigma_{\theta,Rd}}\right) + \left(\frac{\sigma_{\theta,Ed}}{\sigma_{\theta,Rd}}\right)^{k_{i\theta}} + \left(\frac{\tau_{x\theta,Ed}}{\tau_{x\theta,Rd}}\right)^{k_{i\tau}} \leq 1 \tag{9.36}$$

where

$\sigma_{x,Ed}$, $\sigma_{\theta,Ed}$ and $\tau_{x\theta,Ed}$ are the interaction-relevant groups of the significant values of compressive and shear membrane stresses at a single location in the shell segment. The values of the buckling interaction parameters k_{ix}, $k_{i\theta}$, $k_{i\tau}$ and a_i are given in D.4.3.

(4) Where $\sigma_{x,Ed}$ or $\sigma_{\theta,Ed}$ is tensile, its value should be taken as zero in Formula (9.36).

NOTE For axially compressed cylinders with internal pressure (leading to circumferential tension) special provisions are made in Annex D. The resulting value of $\sigma_{x,Rd}$ accounts for both the strengthening effect of internal pressure on the elastic buckling resistance (Formulae (D.55) and (D.56)) and the weakening effect of the elastic-plastic elephant's foot phenomenon (Formulae (D.57) to (D.59)). If the tensile $\sigma_{\theta,Ed}$ is then taken as zero in Formulae (9.33), (9.34), or (9.35), as appropriate, the buckling strength is accurately represented.

(5) The locations and values of each of the buckling-relevant membrane stresses to be used together in combination in Formula (9.36) are defined in D.4.3.

(6) Where the shell buckling condition is not included in Annex D, the buckling interaction parameters may be conservatively estimated using:

$$k_{ix} = 1{,}0 + \chi_x^2 \tag{9.37}$$

$$k_{i\theta} = 1{,}0 + \chi_\theta^2 \tag{9.38}$$

$$k_{i\tau} = 1{,}5 + 0{,}5\ \chi_\tau^2 \tag{9.39}$$

$$a_i = (\chi_x\ \chi_\theta)^2 \tag{9.40}$$

NOTE 1 These rules are sometimes very conservative, but they include the two limiting cases which are well established as safe for a wide range of cases:

a) in very thin shells, the interaction between σ_x and σ_θ is approximately linear;

b) in very thick shells, the interaction becomes that of von Mises.

NOTE 2 The buckling resistance depends very much on the size of the buckles in the buckling mode. Modes dominated by axial compression are very local, those dominated by external pressure are very large, and those dominated by shear lie in between these two sizes.

NOTE 3 This treatment of the interaction between the three membrane stress resultants in different directions is based on analyses of cylindrical shells under the relevant combination with uniform stresses throughout the shell. It is therefore likely to be very conservative where it is applied to a local membrane stress state in a shell (the commonest situation where multiple stresses are all present).

9.6 Design using reference resistances

9.6.1 Principle

(1) Because buckling is not controlled by a single membrane stress at a single location, but depends on the stress state throughout a zone large enough for a buckle to form and which can also include significant plasticity, the buckling limit state, within this sub-clause 9.6, is represented by the design value of the actions, augmented to the point of buckling and applied to the specific defined conditions.

(2) The resistance of the shell should be identified in terms of the parameter R, which is the multiplier on the full set of design loads that leads to the ultimate limit state as defined by the criteria associated with the form of analysis indicated by the subscript following the character.

(3) The influence of membrane and bending effects, and of plasticity and geometric imperfections, are all included by applying the defined values of the capacity parameters to the two reference resistances R_{pl} and R_{cr}.

NOTE The background to the method of Reference Resistance Design is described in Bibliography entries [4] and [6].

9.6.2 Design value of actions

(1) The design values of actions should be taken as in 9.1(1).

9.6.3 Design value of resistance

(1) The design buckling resistance should be determined from the reference elastic critical buckling resistance R_{cr} and the reference plastic resistance R_{pl} for the geometry and load case, together with the capacity parameters α_{s}, β_{s}, η_{s}, λ_{os} and χ_{hs} as defined in Annexes D and E.

(2) The reference plastic resistance R_{pl} is defined in Annexes B, D and E for specific geometries, load cases, and boundary conditions and may only be used for these specific cases.

(3) The value of R_{pl} for a given load case, involving all applied loadings as appropriate (e.g. P_{nEd}, P_{xEd}, p_{nEd}, F_{Ed}, etc.), should be obtained as from Formula (9.41). Where the full loading involves many different load components (multiple forces, pressure distributions etc.), one should be nominated as the leading load F_{Ed} and the ratios between the different loading components should be retained in fixed proportions when identifying the plastic collapse condition. The plastic collapse load of the complete shell assembly F_{Rpl} should then be determined for the magnitude of the leading load. The reference plastic resistance should then be found as the ratio

$$R_{pl} = \frac{F_{Rpl}}{F_{Ed}} \tag{9.41}$$

(4) The elastic critical buckling load F_{Rcr} is defined in Annex E for specific geometries, load cases, and boundary conditions and may only be used for these specific cases.

(5) The reference elastic critical resistance R_{cr} for the same given load case should be obtained from the elastic critical buckling load for the magnitude of the leading load F_{Rcr}. The reference elastic critical resistance should then be found as the following ratio:

$$R_{cr} = \frac{F_{Rcr}}{F_{Ed}} \tag{9.42}$$

(6) The relative slenderness of the shell should be determined from:

$$\overline{\lambda}_s = \sqrt{\frac{R_{pl}}{R_{cr}}} \tag{9.43}$$

(7) The value of the plastic limit relative slenderness $\overline{\lambda}_{s,p}$ should be determined from:

$$\overline{\lambda}_{s,p} = \sqrt{\frac{\alpha_s}{1-\beta_s}} \tag{9.44}$$

$$\alpha_s = \alpha_{s,I}\alpha_{s,G} \tag{9.45}$$

where

$\alpha_{s,G}$ is the geometric reduction factor for the complete shell assembly;

$\alpha_{s,I}$ is the imperfection reduction factor for the complete shell assembly;

β_s is the plastic range factor for the complete shell assembly.

(8) The elastic-plastic buckling reduction factor χ_s should be determined as a function of the relative slenderness of the shell $\overline{\lambda}_s$ from:

$$\chi_s = \chi_{s,h} - \left(\overline{\lambda}_s / \overline{\lambda}_{s,o}\right)\left(\chi_{s,h} - 1\right) \quad \text{when} \quad \overline{\lambda}_s \leq \overline{\lambda}_{s,o} \tag{9.46}$$

$$\chi_s = 1 - \beta_s \left(\frac{\overline{\lambda}_s - \overline{\lambda}_{s,o}}{\overline{\lambda}_{s,p} - \overline{\lambda}_{s,o}} \right)^{\eta_s} \quad \text{when} \quad \overline{\lambda}_{s,o} < \overline{\lambda}_s < \overline{\lambda}_{s,p} \tag{9.47}$$

$$\chi_s = \alpha_s / \overline{\lambda}_s^2 \quad \text{when} \quad \overline{\lambda}_{s,p} \leq \overline{\lambda}_s \tag{9.48}$$

with

$$\eta_s = \left[\frac{\overline{\lambda}_s \left(\eta_{s,p} - \eta_{s,o}\right) + \overline{\lambda}_{s.p}\eta_{s.o} - \overline{\lambda}_{s,o}\eta_{s,p}}{\overline{\lambda}_{s,p} - \overline{\lambda}_{s,o}} \right] \tag{9.49}$$

where

$\overline{\lambda}_{s,o}$ is the squash limit relative slenderness for the complete shell assembly;

$\chi_{s,h}$ is the hardening limit for the complete shell assembly;

$\eta_{s,o}$ is the value of η_s at $\overline{\lambda}_s = \overline{\lambda}_{s,o}$ for the complete shell assembly;

$\eta_{s,p}$ is the value of η_s at $\overline{\lambda}_s = \overline{\lambda}_{s,p}$ for the complete shell assembly.

NOTE The values of these capacity parameters are defined in Annexes D and E.

(9) The characteristic resistance of the shell should be determined from:

$$R_k = \chi_s R_{pl} \tag{9.50}$$

(10) Where $\overline{\lambda}_s \geq \overline{\lambda}_{s,p}$, R_k may be found directly as:

$$R_k = \alpha_I \alpha_G R_{cr} \tag{9.51}$$

(11) The design resistance of the shell should then be determined from:

$$R_d = R_k / \gamma_{M1} \tag{9.52}$$

where

γ_{M1} is the partial factor for resistance of shell to stability as defined in 4.4.

9.6.4 Buckling strength verification

(1) The following verification of the resistance of the shell structure to the defined loading should be undertaken:

$$R_d \geq 1 \tag{9.53}$$

9.7 Design by computational analysis using LBA and MNA analyses

9.7.1 Design value of actions

(1) The design values of actions should be taken as in 9.1 (1).

9.7.2 Design value of resistance

9.7.2.1 General

(1) The design buckling resistance F_{Rd} should be determined from the amplification factor R_d applied to the design values F_{Ed} of the combination of actions for the relevant load case.

(2) The characteristic buckling resistance $F_{Rk} = R_k \cdot F_{Ed}$ should be obtained from the reference plastic resistance $F_{R,pl} = R_{pl} \cdot F_{Ed}$ in combination with the reference elastic critical buckling resistance $F_{R,cr} = R_{cr} \cdot F_{Ed}$. The partial factor γ_{M1} should then be used to obtain the design buckling resistance $F_{Rd} = R_d \cdot F_{Ed}$.

NOTE For the background to the LBA-MNA method, see Bibliography [1] and [2].

9.7.2.2 Reference elastic critical buckling resistance LBA

(1) The reference elastic critical buckling resistance ratio R_{cr} should be determined from an eigenvalue analysis (LBA) applied to the linear elastic calculated stress state in the geometrically perfect shell (LA) under the design values of the load combination. The lowest eigenvalue (bifurcation load factor) should be taken as the elastic critical buckling resistance ratio R_{cr} (see Figure 9.9), but subject to the fuller process given in (3) to (7) below when the shell consists of multiple segments.

(2) It should be verified that the eigenvalue algorithm that is used is reliable at finding the eigenmode that leads to the lowest eigenvalue. In cases of doubt, neighbouring eigenvalues and their eigenmodes should be calculated to obtain a fuller insight into the bifurcation behaviour of the

shell. The analysis should be carried out using software that has been authenticated against benchmark cases with physically similar buckling characteristics.

(3) When conducting an LBA calculation, the complete shell should be considered as an assembly of individual segments, with each segment represented by a simple shell geometry.

(4) It is recommended that, as a first step, the stress design calculations of 9.5 and Annexes D and E are used to obtain a formula-based estimate of both R_{cr} and α for each segment of the complete shell. This may be followed by computationally evaluated R_{cr} (LBA) to which the same value of α is applied. It is then unlikely that an eigenmode that leads to a lower geometrically nonlinear imperfect elastic buckling resistance has not been detected.

NOTE The notation α used here is intended to relate to the product $\alpha_G\alpha_I$ according to Annexes D and E.

(5) Where the above process does not convincingly identify an appropriately low value of αR_{cr}, a sufficient number of different eigenvalues, each giving different values of R_{cr}, and potentially relating to segments with different geometries, should be calculated (progressively rising from the lowest eigenvalue) and their corresponding eigenmodes examined to ensure that the globally lowest αR_{cr} has been identified. In this case, the same chosen value of α for each segment should be adopted as in (4).

NOTE The above procedure covers the possibility that the different eigenvalues may be for buckling modes that occur in shell segments that have different thicknesses or lengths, so that α for each segment may be different, potentially leading to a lower αR_{cr} for a segment that does not have the lowest eigenvalue.

(6) The LBA buckling modes should always be examined to identify their location and character (local or global). This information should also be used in 9.7.2.4 (4) where the relevant buckling parameters are to be chosen.

NOTE Where the critical LBA mode and the MNA plastic collapse mode occur in different segments or locations and the corresponding resistances R_{cr} and R_{pl} are used in combination to find R_k, the LBA-MNA procedure gives a safe estimate of the correct elastic-plastic resistance R_k (see Bibliography [2]).

(7) For each segment, the local value for the elastic imperfection reduction factor, termed $\alpha_{s,LM}$ should be determined using relevant formulae given in Annexes D and E. In interpreting this paragraph, the segment can be seen as having constant wall thickness, or can be of stepped construction in which multiple strakes have different wall thicknesses, or can be between boundaries or stiffening rings, depending on the specific conditions.

(8) The corresponding assessed elastic imperfect buckling resistance $\alpha_{s,LM}R_{cr}$ should be found for each segment. The segment that has the lowest value of $\alpha_{s,LM}R_{cr}$ should be deemed to be the critical segment for elastic buckling. This elastic critical buckling resistance R_{cr} and elastic imperfection reduction factor $\alpha_{s,LM}$ should then be adopted as the relevant values for the complete shell.

NOTE 1 Under axial compressive stresses, the buckling mode is local and usually confined to a zone of constant wall thickness, but under external pressure in stepped wall construction, the buckling mode often covers multiple strakes of different thickness. The interpretation of the above paragraph depends on the conditions in the particular structure, so consideration is needed to achieve a useful interpretation.

NOTE 2 A multi-strake wind turbine support tower is often an assembly of individual cylindrical and truncated conical shell segments, while a pressure vessel can be an assembly of cylindrical and spherical shell segments. In some cases, the procedure defined in (5) and (6) requires very many eigenmodes to be calculated, depending on the complexity of the structure, since a shell segment that is not the most critical can still exhibit a large number of modes with similar eigenvalues.

9.7.2.3 Reference plastic resistance MNA

(1) The reference plastic resistance R_{pl} (see Figure 9.9) should be obtained using the ideal elastic-plastic stress strain curve in a materially nonlinear analysis (MNA) as the plastic limit load under the applied combination of actions. This load ratio R_{pl} may be taken as the largest value attained in the analysis, using an ideal elastic-plastic material law.

NOTE Improved methods for accurate evaluation of the plastic limit load are available, which permit progressively better estimates of the true value of R_{pl} to be obtained without the analysis having to approach the plateau closely (see Bibliography [3] and [6]).

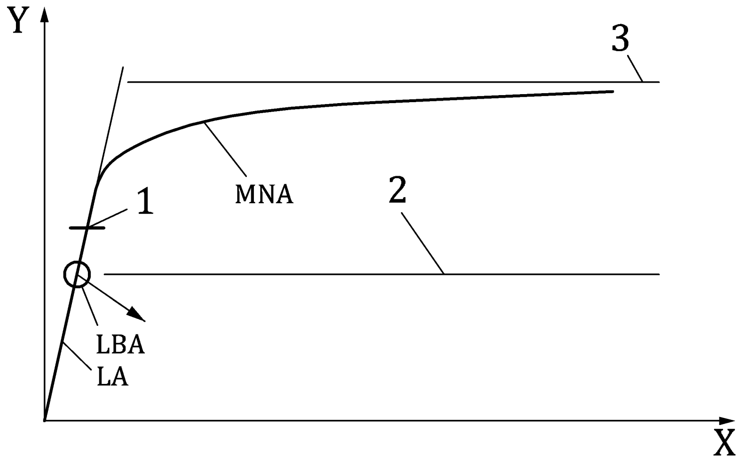
(2) A GMNA analysis may only be used to establish a value for the reference plastic resistance R_{pl} if an ideal elastic-plastic stress strain curve is used.

NOTE Where the shell displays geometric softening, the GMNA analysis will give a safe estimate of the MNA outcome for R_{pl}. Where the shell displays geometric hardening, the GMNA analysis will overestimate the strict MNA result, but the use of the deduced value of R_{pl} is not unsafe unless the plastic collapse mode and the imperfect elastic buckling mode are in different locations (see Bibliography [2]).

(3) When conducting the MNA calculation, the complete shell should be considered as an assembly of individual segments, with each segment represented by a simple shell geometry (e.g. cylinder, cone, sphere, toroid etc.). The plastic collapse mechanism should be examined to determine the location and form of the collapse mode and its location relative to the LBA mode. The segment that exhibits the largest plastic strains or displacements should be taken as the critical segment for plastic collapse and the corresponding resistance as R_{pl}.

NOTE 1 It is possible that the location of the plastic collapse mode found in this way can be different from the location of the most imperfect buckling mode. However, if the reference plastic resistance R_{pl} found in the above manner is adopted in the complete shell buckling assessment, the conservatism of the outcome is secure if it is appropriately combined with the lowest imperfect elastic buckling resistance αR_{cr}, even when the imperfect elastic or elastic-plastic buckling mode is in a different part of the shell (see Bibliography [1] and [2]).

NOTE 2 Where the shell has only one segment, many of the issues identified in the following paragraphs do not apply and the complete process can be undertaken without the checks described.



Key

X Deformation

Y Load factor on design actions R

1 R_{pl} estimated from LA

2 R_{cr} from linear elastic bifurcation

3 R_{pl} small displacement theory plastic limit load

Figure 9.9 — Definition of plastic reference plastic resistance R_{pl} and reference elastic critical buckling resistance R_{cr} derived from MNA and LBA analyses respectively

NOTE 3 Figure 9.9 is shown only as an illustration of different potential load-deformation paths that a shell structure can display. For slender structures ($\overline{\lambda} \geq \overline{\lambda}_p$), the relative magnitudes in Figure 9.9 are relevant. The falling arrow for post-buckling after LBA is used to illustrate the dangerous post-buckling response of slender shells under common loadings. For stocky structures ($\overline{\lambda} \leq \overline{\lambda}_p$), the value of the LBA (R_{cr}) will be much higher than the value of the MNA (R_{pl}). A single figure is shown to cover all analysis types, but the relative magnitudes are not appropriate to all structures.

(4) Where it is not possible to undertake a materially nonlinear analysis (MNA), the reference plastic resistance ratio R_{pl} may be conservatively estimated from linear shell analysis (LA) conducted using the design values of the applied combination of actions using the following procedure. The evaluated membrane stress resultants $n_{\mathrm{x,Ed}}$, $n_{\theta,\mathrm{Ed}}$ and $n_{\mathrm{x}\theta,\mathrm{Ed}}$ at any point in the shell should be used to estimate the reference plastic resistance from:

$$R_{pl} = \frac{t \cdot f_{yk}}{\sqrt{n_{x,Ed}^2 - n_{x,Ed} \cdot n_{\theta,Ed} + n_{\theta,Ed}^2 + 3n_{x\theta,Ed}^2}} \tag{9.54}$$

(5) The lowest value of plastic resistance ratio calculated using either (1) or (4) should be taken as the estimate of the reference plastic resistance ratio R_{pl}.

NOTE 1 A safe estimate of R_{pl} can usually be obtained from an LA analysis as follows. The three points in the shell are identified, for the three places where each of the three buckling-relevant membrane stress resultants attains its highest value. Formula (9.54) is then applied separately to the stress state at each of these three points to obtain three separate estimates of R_{pl}. The relevant estimated value of R_{pl} can then be taken as the lowest of these three estimates.

NOTE 2 In many cases, Formula (9.54) provides a rather low estimate of R_{pl}. However, where shell bending dominates the structural behaviour (e.g. in flat discs), this estimate can be unconservative.

9.7.2.4 Elastic-plastic resistance assessment

(1) Although the complete shell system may be treated as a single unit, it may be useful to consider each segment separately using the full LBA-MNA treatment to ensure that it meets the local resistance requirements.

NOTE Where the critical LBA mode and the MNA plastic collapse mode occur in different segments or locations and the corresponding reference resistances R_{cr} and R_{pl} are used in combination to find R_k, the LBA-MNA procedure gives a safe estimate of the correct elastic-plastic resistance R_k (see Bibliography [2]).

(2) The relative slenderness $\bar{\lambda}_s$ for the complete shell assembly should be determined from:

$$\bar{\lambda}_s = \sqrt{F_{\mathrm{R,pl}} / F_{\mathrm{R,cr}}} = \sqrt{R_{\mathrm{pl}} / R_{\mathrm{cr}}} \tag{9.55}$$

(3) The complete shell elastic-plastic buckling reduction factor χ_s should be determined as $\chi_{\mathrm{s}} = f\left(\bar{\lambda}_{\mathrm{s}}, \chi_{s,h,LM}, \bar{\lambda}_{\mathrm{s,0,LM}}, \alpha_{\mathrm{s,LM}}, \beta_{\mathrm{s,LM}}, \eta_{\mathrm{s,LM}}\right)$ using 9.6.3 (8), to obtain the lowest value for the complete structural system. Here the parameters for the complete shell are found: $\alpha_{s,LM}$ is the adopted elastic imperfection reduction factor, $\beta_{s,LM}$ is the adopted plastic range factor, $\eta_{s,LM}$ is the adopted interaction exponent, $\bar{\lambda}_{\mathrm{s,0,LM}}$ is the adopted squash limit relative slenderness and $\chi_{s,h,LM}$ is the adopted shell hardening limit.

NOTE The notation LM associated with each of the adopted parameters is used to indicate that this is a value adopted for the LBA-MNA method and assessed by comparison of geometries, loading and eigenmodes.

(4) The values for the factors $\bar{\lambda}_{\mathrm{s,0,LM}}$, $\alpha_{s,LM}$, $\beta_{s,LM}$, $\eta_{s,LM}$ and $\chi_{s,h,LM}$ should be determined by comparison with known shell buckling cases identified in Annexes D and E that have similar buckling modes, similar imperfection sensitivity, similar geometric nonlinearity, similar yielding sensitivity, similar hardening and similar post-buckling behaviour where each of these aspects is relevant. The value of $\alpha_{s,LM}$ should also take account of the appropriate fabrication tolerance quality class.

(5) Two separate sets of capacity parameters $\lambda_{s,0,LM}$, $\alpha_{s,LM}$, $\beta_{s,LM}$, $\eta_{s,LM}$ and $\chi_{s,h,LM}$ should be adopted. The first should be applied to the segment identified as critical for plastic collapse in 9.7.2.3, and the second should be applied to the segment identified as critical for elastic buckling in 9.7.2.2. A consistent treatment should be applied for each segment individually using the geometry of that particular segment. These two sets of factors should then be used to provide two separate estimates for χ_S. The lower estimate for χ_S should be retained in the design evaluation. If the same segment is critical for both cases, only one estimate is obtained and is sufficient.

(6) Where the shell assembly represents a long cylindrical structure such as a wind turbine support tower, the value of the shell length used in the expressions in Annexes D and E should be taken as the length between locations where the cross-section is maintained as circular.

(7) Special care should be taken when choosing an appropriate value of $\alpha_{s,LM}$ if this approach is used on shell geometries and loading cases in which significant geometric pre-buckling nonlinearity or snap-through buckling can occur, since $\alpha_{S,LM}$ does not distinguish between α_G which is unaffected by fabrication quality and α_I which depends on the imperfection amplitude and therefore the fabrication quality.

NOTE 1 Snap-through buckling can occur in conical and spherical caps and domes under external pressure or on supports that can displace radially. Similar care is needed in choosing an appropriate value of $\alpha_{s,LM}$ when the shell geometry and load case produce conditions that are highly sensitive to changes of geometry, such as cylinders under global bending where ovalisation can occur and unstiffened junctions between cylindrical and conical shell segments under meridional compressive loads (e.g. in chimneys).

NOTE 2 Commonly reported elastic shell buckling loads in the literature for these special cases are normally based on geometrically nonlinear analysis applied to a perfect or imperfect geometry, which consequently sometimes mix predictions of the snap-through buckling load with those attributed to imperfections. By contrast, the methodology used here adopts the linear bifurcation load as the reference elastic critical buckling resistance. This is often much higher than the snap-through load for shallow caps and domes.

(8) The design calculation should take account of these two sources of reduced resistance by an appropriate choice of the complete shell elastic imperfection reduction factor $\alpha_{s,LM}$. This choice should include the effect of both the geometric nonlinearity (that can lead to snap-through) and the additional strength reduction caused by geometric imperfections.

(9) If the provisions of (3) or (4) cannot be achieved beyond reasonable doubt, appropriate GMNIA calculations should be undertaken, according to 9.8. Alternatively, tests should be carried out, see prEN 1990:2021, Annex D, though these should be designed and interpreted with care, since laboratory tests do not easily reproduce the geometric imperfections found in the final construction.

(10) If specific values for the capacity parameters $\alpha_{s,LM}$, $\beta_{s,LM}$, $\eta_{s,LM}$, $\overline{\lambda}_{s,0,LM}$ and $\chi_{s,h,LM}$ are not obtainable according to (4) or (5), and buckling is expected to be in the elastic domain ($\overline{\lambda}_s \geq \overline{\lambda}_{s,p}$), the values for an axially compressed unstiffened cylinder may be adopted, see D.3.3.3. Where the same situation occurs with $\overline{\lambda}_s < \overline{\lambda}_{s,p}$, the values for an unstiffened cylinder under global bending should be used, see E.3.2.4. Where snap-through is known to be a possibility, appropriate further reductions in $\alpha_{s,LM}$ should be considered, associated only with a reduction in α_G.

(11) The characteristic buckling resistance ratio R_k should be obtained from:

$$R_k = \chi_s R_{pl} \tag{9.56}$$

where

R_{pl} is the reference plastic resistance ratio.

(12) The design buckling resistance ratio R_d should be obtained from:

$$R_d = R_k/\gamma_{M1} \tag{9.57}$$

where

γ_{M1} is the partial factor for resistance of shell to stability as defined in 4.4.

9.7.3 Buckling strength verification

(1) It should be verified that:

$$F_{Ed} \leq F_{Rd} = R_d \cdot F_{Ed} \quad \text{or} \quad R_d \geq 1 \tag{9.58}$$

9.8 Design by computational analysis using GMNIA analysis

9.8.1 Design values of actions

(1) The design values of actions should be taken as in 9.1 (1).

9.8.2 Design value of resistance

(1) The design buckling resistance should be determined as a load factor R applied to the design values F_{Ed} of the combination of actions for the relevant load case.

(2) The characteristic buckling resistance ratio R_k should be found from the imperfect elastic-plastic buckling resistance ratio R_{GMNIA}, adjusted by the calibration factor k_{GMNIA} where necessary. The design buckling resistance ratio R_d should then be found using the partial factor γ_{M1}.

(3) To determine the imperfect elastic-plastic buckling resistance ratio R_{GMNIA}, a GMNIA analysis of the geometrically imperfect shell under the applied combination of design values of actions should be carried out, accompanied by automated tests along the load path for possible bifurcations.

NOTE Where plasticity has a significant effect on the buckling resistance, an adopted imperfection mode is needed in which some pre-buckling shear strains exist, because the shear modulus is very sensitive to small plastic shear strains. In certain shell buckling problems (e.g. shear buckling of annular plates), if this effect is omitted, the eigenvalue analysis can give a considerable overestimate of the elastic-plastic buckling resistance.

(4) An LBA analysis should first be performed on the perfect structure to determine the reference elastic critical buckling resistance ratio R_{cr} of the perfect shell.

(5) The LBA critical buckling mode should be examined to identify its location and character (local or global).

(6) An MNA analysis, adopting a perfect elastic-plastic material representation, should next be performed on the perfect structure to determine the reference perfect plastic resistance ratio R_{pl}.

(7) The MNA plastic mechanism should be examined to check its form (bending or rupture being dominant) and its location relative to the LBA critical mode.

(8) The LBA and MNA resistance ratios should then be used to establish the complete shell relative slenderness $\overline{\lambda}_s$ according to Formula (9.55).

(9) A GMNA analysis should then be performed on the perfect structure to determine the perfect elastic-plastic buckling resistance ratio R_{GMNA}. This resistance ratio should be used later to verify that the effect of the chosen geometric imperfections has a sufficiently deleterious effect to give confidence that the lowest resistance has been obtained. The GMNA analysis should be carried out under the applied combination of actions, accompanied by automated tests along the load path for possible bifurcations.

(10) When GMNA, GMNIA and GNIA analyses are used, eigenvalue checks should always be performed throughout the load path to ensure that any possible bifurcation is detected.

(11) The imperfect elastic-plastic buckling resistance ratio R_{GMNIA} should be found as the lowest load factor R obtained from the three following criteria C1, C2 and C3, see Figure 9.10:

— Criterion C1: The maximum load factor on the load-deformation-curve (limit load);

— Criterion C2: The bifurcation load factor, where this occurs during the loading path before reaching a limit point on the load-deformation-curve;

— Criterion C3: The largest tolerable deformation, where this occurs during the loading path before reaching a bifurcation load or a limit load.

NOTE 1 Criterion C3 can seem to be a serviceability restriction rather than a safety critical ultimate limit state, but it is a helpful criterion to avoid excessive deformations in the structure in service.

NOTE 2 Criterion C3 can apply to structures whose behaviour is entirely elastic but which are susceptible to very large deformations (e.g. the inversion of a cylinder without a ring stiffener at its end).

(12) The largest tolerable deformation should be assessed relative to the conditions of the individual structure. If no other value is available, the largest tolerable deformation may be deemed to have been reached when the greatest local rotation of the shell surface (slope of the surface relative to its original geometry) attains the value β_{lim}.

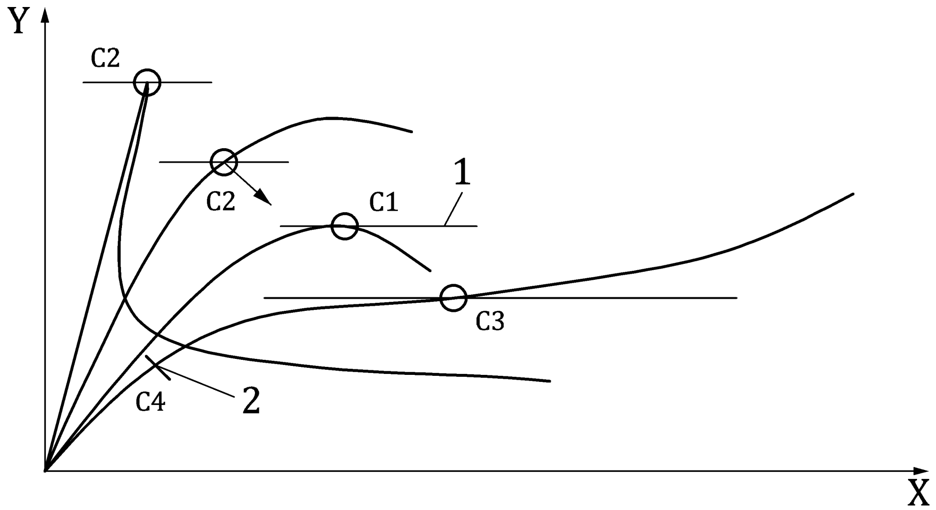
NOTE The value of β_{lim} is taken as β_{lim} = 0,1 radians, unless the National Annex gives a different value.

(13) An alternative test, where the structure experiences considerable yielding, is to use the plastic strain limitation of 7.3 (4) to assess Criterion C3.

(14) Where a bifurcation is detected in the GMNA analysis, but the GMNIA analysis passes smoothly from pre-buckling to post-buckling, the GMNA bifurcation resistance should initially be taken as the ultimate value according to Criterion C3.

(15) Where the GMNIA analysis passes smoothly from pre-buckling to post-buckling, the GMNIA analysis should be repeated with an imperfection of amplitude of 20% of the previously chosen value to determine whether a small imperfection is more susceptible to buckling than both the perfect shell (GMNA) and GMNIA with the previously assumed imperfection amplitude. Consideration should also be given to the procedure defined in (35).

NOTE The phenomenon of a structure with a significant imperfection passing smoothly from pre-buckling to post-buckling is quite common (Bibliography [5] and [6]), but the constructed shell is likely to have smaller imperfections than those assumed in the GMNIA analysis, so in this case it is advisable to note the bifurcation in GMNA as the first measure of buckling resistance.



Key

X Deformation

Y Load factor on design actions R

1 $R_{\text{GMINA}}$is the lowest of these alternative measures

2 First yield safe estimate

Figure 9.10 — Definition of buckling resistance from a GMNIA analysis

(16) Using the Criterion C4, a conservative assessment of the imperfect elastic-plastic buckling resistance ratio R_{GMNIA} may be obtained using an elastic GNIA analysis of the geometrically imperfect shell under the applied combination of actions. The lowest load factor $R_{\text{GNIA.y}}$ should be obtained according to Criterion C4, unless a bifurcation has been detected earlier

— Criterion C4: The load factor $R_{\text{GNIA.y}}$ at which the equivalent stress at the most highly stressed point on the shell surface reaches the design value of the yield stress $f_{\text{yd}} = f_{\text{yk}}/\gamma_{\text{M0}}$, see Figure 9.10.

NOTE Criterion C4 is very conservative since it relates only to first yield on the surface. It could alternatively be used with the equivalent stress determined from membrane stress resultants.

(17) In formulating the GMNIA (or GNIA) analysis, appropriate allowances should be considered for incorporation into the model to cover the effects of imperfections that cannot be avoided in practice, including:

a) geometric imperfections, such as:

— deviations from the nominal geometric shape of the middle surface (pre-deformations, out-of-roundness);

— irregularities at and near welds (minor eccentricities, minor misalignments, shrinkage depressions, rolling curvature errors);

— deviations from nominal thickness;

— lack of evenness of supports.

b) material imperfections, such as:

— residual stresses caused by rolling, pressing, welding, straightening etc.;

— inhomogeneities and anisotropies.

NOTE 1 Further possible negative influences on the imperfect elastic-plastic buckling resistance ratio R_{GMNIA}, such as ground settlements or flexibilities of connections or supports, are not classed as imperfections in the sense of these provisions.

NOTE 2 Many of the items on the above list have not been extensively researched, and some play only a very minor role. The list is provided to encourage designers to consider all plausibly significant influencing factors in so far as is possible.

(18) Imperfections should be allowed for in the GMNIA (or GNIA) analysis by including appropriate additional quantities in the analytical model for the numerical computation.

(19) The imperfections should generally be introduced by means of equivalent geometric imperfections in the form of initial shape deviations perpendicular to the middle surface of the perfect shell, unless a better technique is used. The middle surface of the geometrically imperfect shell should be obtained by superposition of the equivalent geometric imperfections on the perfect shell geometry.

(20) The pattern of the equivalent geometric imperfections should be chosen in such a form that, under the defined loading condition, it has the most unfavourable effect on the imperfect elastic-plastic buckling resistance ratio R_{GMNIA} of the shell.

(21) The information given in 9.4 on the relevance of different forms of imperfection to stress states in the shell should be used to inform the choice of imperfection form.

(22) If the most unfavourable pattern cannot be readily identified beyond reasonable doubt, the analysis should be carried out for a sufficient number of different imperfection patterns, and the worst case (lowest value of R_{GMNIA}) should be identified.

(23) The pattern of the equivalent geometric imperfections should, if practicable, reflect the constructional detailing and the boundary conditions in an unfavourable manner.

(24) Imperfection patterns that have been demonstrated to be severe for shell buckling in relation to the shell geometry and loading conditions may be taken to be sufficient.

(25) Modification of the adopted mode of geometric imperfections to include realistic structural details (such as axisymmetric weld depressions) should be explored.

NOTE Where axial compression dominates in considerations for design against buckling, the very local nature of the weld depression, coupled with its extremely severe effect on buckling resistance even when not extending around a large part of the shell, indicate that it can be the most detrimental imperfection form in welded structures.

(26) The eigenmode-affine pattern may be used unless a different more unfavourable pattern is clearly relevant (see (23)-(25), (36) and (37)).

NOTE 1 The eigenmode affine pattern is the critical buckling mode associated with the reference elastic critical buckling resistance ratio R_{cr} based on an LBA analysis of the perfect shell under the defined loading condition.

NOTE 2 The eigenmode-affine pattern can also be far from the most unfavourable pattern.

NOTE 3 When choosing suitable imperfection patterns normal to the shell mid-surface as defined in (19), due consideration is required of all potentially relevant patterns, which include eigenmode affine patterns (as in (24)), weld depression patterns, collapse affine patterns and post-buckling affine patterns. These patterns can be derived from GNA or GMNA calculations and they can, in particular circumstances, lead to lower resistance evaluations.

(27) Where eigenmode imperfections are adopted and the shell consists of multiple segments (whether through change of thickness or change of shell geometry), a sufficient number of LBA eigenmodes should be extracted and examined to ensure that a buckling mode has been found that will give the lowest value of αR_{cr} for the structure. Where the estimated value of α is similar for different segments, the search for a higher eigenvalue that leads to a lower imperfect elastic buckling resistance is not required to be extensive. But where the shell consists of different segments that can have significantly different imperfection sensitivity, higher eigenvalues should be explored until the product of potential imperfection sensitivity α and critical resistance R_{cr} exceeds the previously established lowest such product. Each calculated eigenmode should then be investigated to identify the globally critical geometric imperfection.

NOTE Shear buckling modes are not very sensitive to imperfection forms that occur in typical fabricated shells.

(28) Equivalent geometrical imperfections that are parallel to the shell middle surface (introducing membrane forces) should also be considered where appropriate (e.g. imperfections of the bottom face of a vertical cylindrical shell).

(29) Equivalent geometrical imperfections in the form of boundary unevenness (e.g. imperfections of the bottom face of a vertical cylindrical shell) should be considered where this can occur in the construction.

NOTE 1 EN 1090-2 does not specify imperfection patterns, and only defines measuring gauge and amplitudes which relate directly to Formula (9.59) only for conditions of uniform axial compression.

NOTE 2 Further appropriate patterns of imperfections can be considered by the designer based on available authoritative research when the design is being verified.

(30) Notwithstanding (19) to (29), patterns may be excluded from the investigation if they can be eliminated as unrealistic because of the method of fabrication, manufacture or erection.

NOTE For example, eigenmode imperfections relating to shear or torsional buckling modes are not commonly found in fabricated shells, so modes of this kind can be adopted at a lower amplitude or set aside as improbable.

(31) The sign of the equivalent geometric imperfections should be chosen in such a manner that the maximum initial shape deviations are unfavourably oriented towards the centre of the shell curvature.

(32) The amplitude of the adopted equivalent geometric imperfection form should be interpreted in a manner consistent with the tolerance measurements defined in 9.4.5. To achieve this when using a calculated eigenmode, post-buckling mode or collapse-affine mode, an appropriate calibration must be undertaken using a notional measuring gauge to deduce the relationship between the peak deformation of the mode (1,0) and the magnitude that would be measured by the tolerance measurement (usually > 1,0), see Figure 9.4.

(33) The amplitude of the adopted equivalent geometric imperfection form should be taken as dependent on the fabrication tolerance quality class. The deviation of the geometry of the equivalent imperfection from the perfect shape $\delta_{0,eq}$ should be taken as:

$$\delta_{0,eq} = \ell_g \, U_n \tag{9.59}$$

where

ℓ_g is all relevant gauge lengths according to 9.4.5 (4);

t is the local shell wall thickness;

U_n is the dimple imperfection amplitude parameter for the relevant fabrication tolerance quality class.

The value of the dimple tolerance parameter U_n is given in Table 9.5.

Table 9.5 — Values for the dimple imperfection amplitude parameter U_n

Fabrication tolerance quality class	Description	Value of $\boldsymbol{U_n}$
Class A	Excellent	0,006
Class B	High	0,010
Class C	Normal	0,016

NOTE 1 The above values are linked to the dimple tolerance measurements for axial compression together with the assumed imperfection amplitudes for simple calculation (Formulae (D.11) to (D.13)). This alignment means that GMNIA calculations using these amplitudes are expected to reproduce the formulae of D.1.3 quite closely.

NOTE 2 Only dimple imperfections are considered in the above requirements. Since a shell can have other imperfection forms in addition to the modelled dimples this procedure is potentially unsafe. However, these calculations are based on a full circumference of an axisymmetric dimple (the worst case) which can rarely occur in practice, the full tolerance amplitude is rarely attained, and the potential for a practical load case to induce the resistance stress at the critical locations is small. For these reasons, the omission of an additional margin to allow for other imperfections is generally not serious.

(34) The amplitude of the geometric imperfection in the adopted pattern of the equivalent geometric imperfection should be interpreted in a manner which is consistent with the gauge length method, set out in 9.4.5 (4), by which it is defined. The gauge of length $25t$ (Formula (9.10)) is not required to be used in these calculations.

(35) Additionally, it should be verified that an analysis that adopts an imperfection whose amplitude is 10% smaller than the value $\delta_{0,eq}$ found in (33) does not yield a lower value for the ratio R_{GMNIA}. If a lower value is obtained, the procedure should be repeated to find the lowest value of the ratio R_{GMNIA} as the amplitude is varied.

(36) Where the construction form is more susceptible to other imperfection forms than dimples, the most damaging imperfection form should be explored with magnitudes corresponding to the tolerance limits for the relevant fabrication quality tolerance class. The outcomes of GMNIA analyses using this form should be compared with the simple calculation predictions of resistance in Annex D to verify that the procedures of Annex D are still valid for design purposes.

(37) If follower load effects are possible, either they should be incorporated in the analysis, or it should be verified that their influence is negligible.

(38) For each calculated value of the imperfect elastic-plastic buckling resistance R_{GMNIA}, the ratio of the imperfect to perfect resistance (R_{GMNIA}/R_{GMNA}) should be determined and compared with values of α found using the procedures of 9.5 and Annexes D and E, to verify that the chosen geometric imperfection has a deleterious effect that is comparable with those obtained from test results.

NOTE Where the resistance is dominated by plasticity effects, the ratio (RGMNIA/RGMNA) will be much larger than the elastic buckling reduction factor α, and no close comparison can be expected. However, where the resistance is controlled by buckling phenomena that are substantially elastic, the ratio (RGMNIA/RGMNA) is expected to be only slightly higher than the value determined by simple calculation, and the features that have led to any substantially higher value require careful consideration.

(39) The reliability of the numerically determined imperfect elastic-plastic buckling resistance ratio R_{GMNIA} should be checked by one of the following alternative methods:

a) by using the same program to calculate values $R_{\mathrm{GMNIA,check}}$ for other shell buckling cases for which characteristic buckling resistance ratio values $R_{\mathrm{k,known}}$ are known. The check cases should use comparable imperfection assumptions and be similar in their buckling controlling parameters (such as relative shell slenderness, post-buckling behaviour, imperfection-sensitivity, geometric nonlinearity and material behaviour);

b) by comparison of calculated values ($R_{\mathrm{GMNIA,check}}$) against test results ($R_{\mathrm{test,known}}$). The check cases should satisfy the same similarity conditions given in (a).

NOTE 1 Other shell buckling cases for which the characteristic buckling resistance ratio values $R_{\mathrm{k,known}}$ are known can be found from the scientific literature on shell buckling. Some of the simple calculation provisions of Annex D have been derived as general lower bounds on test results, but these sometimes lead to such low assessed values for the characteristic buckling resistance that they cannot be easily reproduced numerically.

NOTE 2 The rules of this standard for buckling under uniform axial compression (D.3.3) and uniform bending (E.3.2) are potentially useful as benchmark predictions by GMNIA since they have been very thoroughly verified. However these algebraic rules were obtained by fitting approximate expressions to the numerical data, so they could not be perfectly reproducible by GMNIA.

(40) Where test results are used, it should be established that the geometric imperfections present in the test are expected to be representative of those that will occur in practical construction.

(41) Where the reliability check is performed using a well-established known resistance established in the literature, the calibration factor k_{GMNIA} should be evaluated using:

$$k_{\mathrm{GMNIA}} = \frac{R_{\mathrm{k,known}}}{R_{\mathrm{GMNIA,check}}} \tag{9.60}$$

where

$R_{k,known}$ is the known characteristic value;

$R_{GMNIA,check}$ is the calculation prediction of the known buckling case.

(42) Where a known characteristic value based on existing established theory is used to determine k_{GMNIA}, and the calculated value of k_{GMNIA} lies outside the range $0{,}8 < k_{GMNIA} < 1{,}2$, this procedure should not be used. The GMNIA result should be deemed invalid, and further calculations undertaken to establish the causes of the discrepancy.

(43) Where the reliability check is performed using well established test data, the calibration factor k_{GMNIA} should be evaluated using:

$$k_{\mathrm{GMNIA}} = \frac{R_{\mathrm{test}}}{R_{\mathrm{GMNIA,check}}} \tag{9.61}$$

where

R_{test} is the well established test result;

$R_{GMNIA,check}$ is the calculation outcome when predicting the test.

(44) Where test results are used to determine k_{GMNIA}, and the calculated value of k_{GMNIA} exceeds 1,0, the adopted value should be $k_{\mathrm{GMNIA}} = 1{,}0$.

NOTE The formulae given in D.3.3 and E.3.2 can be used to provide benchmarking calculations classed as existing theory.

(45) The characteristic buckling resistance ratio should be obtained from:

$$R_{\mathrm{k}} = k_{GMNIA}\, R_{GMNIA} \tag{9.62}$$

where

R_{GMNIA} is the calculated imperfect elastic-plastic buckling resistance ratio;

k_{GMNIA} is the calibration factor.

9.8.3 Buckling strength verification

(1) The design buckling resistance ratio R_{d} should be obtained from:

$$R_{\mathrm{d}} = R_{\mathrm{k}}/\gamma_{\mathrm{M1}} \tag{9.63}$$

where

γ_{M1} is the partial factor for resistance of shell to stability as defined in 4.4.

(2) It should be verified that:

$$F_{Ed} \le F_{Rd} = R_d \cdot F_{Ed} \quad \text{or} \quad R_d \ge 1 \tag{9.64}$$

10 Fatigue Limit State (LS4)

10.1 Design values of actions

(1) The design values of the actions for each load case should be taken as the varying parts of the total action representing the anticipated action spectrum throughout the design life of the structure.

(2) Design values of actions and load spectra that produce stress ranges $\Delta\sigma$ relevant to the fatigue limit state may be specified in EN 1991, in application parts of EN 1993 or in relevant product specifications.

(3) If equivalent constant stress ranges $\Delta\sigma_{\mathrm{e,2,Ed}}$ as defined in prEN 1993-1-9:2023, 7.3.2 or 7.3.3 are specified in the documents identified in (2), it should be verified that their definition matches the chosen stress design approach (see 10.2). In other cases, the design value of an equivalent stress range $\Delta\sigma_{\mathrm{e,2,Ed}}$ may be calculated according to prEN 1993-1-9:2023, 7.3.4 from the linearly accumulated damage D at the relevant construction detail. The cumulative linear damage model of prEN 1993-1-9:2023, Annex A should be used to calculate D.

10.2 Stress design

10.2.1 General

(1) The stress ranges $\Delta\sigma$ and the stress range spectra resulting from fatigue actions should be calculated at relevant constructional details or notches in the shell, considering the appropriate design stress methods of prEN 1993-1-9:2023, 6.1(1). Fatigue actions are loading events whose number of occurrences may cause fatigue, see prEN 1993-1-9:2023, 3.1.2.

NOTE Constructional details relevant to LS4 are generally found at welded or bolted joints, connections, stiffeners or attachments. A classification of constructional details and the corresponding fatigue resistance values to be used for the chosen design stress method are given in prEN 1993-1-9:2023, Clause 10, Annex B and Annex C for the nominal or modified nominal stress method, hot spot stress method and effective notch stress method, respectively.

(2) The nominal stress method of EN 1993-1-9 may be used in cases where membrane stresses in the shell middle surface represent the design stress range spectrum in the proximity of the considered notch with sufficient accuracy. This can be assumed to be the case for long cylindrical shells with $r/t \leq 300$ that are primarily under meridional and membrane shear loading, such as masts, towers and chimneys, which can usually be treated by simple beam theory. In this case, the application of stress concentration factors k_{f} or $k_{\mathrm{f,imp}}$ as defined in 10.2.2 (5) and Table 10.1 or in prEN 1993-1-9:2023, Annex D may still be required to account for intended and unintended eccentricities at joints between plates.

(3) In cases where closed-form formulae, such as those given in Annex C, are used for the shell stress analysis, the resulting stresses should be seen as modified nominal stresses as described in prEN 1993-1-9:2023, 7.3.2 (2).

(4) In cases that cannot be treated by the analysis methods of (2) and (3), the hot spot stress method of prEN 1993-1-9:2023, Annex B or the effective notch stress approach of prEN 1993-1-9:2023, Annex C should be used for fatigue design of shell structures.

10.2.2 Stress calculation methods

(1) Stresses used for verifications in the fatigue limit state (LS4) should be determined using the design values of the fatigue actions and an appropriate method of stress analysis for the considered shell and constructional detail:

— membrane theory (or simple beam theory in the case of long, thick-walled cylindrical shells) for shells and constructional details that can be treated using the nominal stress method, see 10.2.1 (2);

— closed-form formulae derived from shell bending theory, such as those given in Annex C. These stresses should be interpreted as modified nominal stresses, as defined in prEN 1993-1-9:2023, 7.3.1 (2);

— computational LA or GNA analysis, subject to the provisions in (2) to (6). Depending on the modelling and discretization, these stresses can be structural or effective notch stresses as defined in EN 1993-1-9.

(2) In identifying the resistance to fatigue, intended joint eccentricities (Figure 9.6 (b)) should always be taken into account in the stress calculation, either by appropriate stress concentration factors k_{f} (see Annex C.3 (2) or prEN 1993-1-9:2023, Annex D) or by direct modelling of an offset in the shell middle surface in an LA or GNA analysis.

(3) Secondary stresses caused by shell boundary conditions and changes in loadings should always be taken into account (see Annex C), with the exception of long thick-walled cylindrical shells (see 10.2.1 (2)).

(4) The magnitude of unintended joint eccentricities (see 9.4.4) at constructional details required for the applicability of the fatigue classes and resistances given in EN 1993-1-9 should be considered. Corresponding requirements for the application of the fatigue resistances are given in EN 1993-1-9.

NOTE The direct applicability of the fatigue resistances in EN 1993-1-9 generally requires that the execution conforms to EN 1090-2 EXC3 for welded structures and that undesired eccentricities or misalignments between plates or shell walls at constructional details with butt welds are smaller than 5% of the wall thickness.

(5) If the requirements in (4) cannot be specified in design or met in fabrication, the unintended eccentricity should either be explicitly included in the stress analysis, or the stresses calculated without explicit consideration of unintended eccentricities should be modified by multiplying the membrane stress component that is perpendicular to the orientation of the considered eccentricity with the stress concentration factors $k_{\mathrm{f,imp}}$ of Table 10.1. In this table, the factors $k_{\mathrm{f,imp}}$ depend on the

stress calculation method and the fabrication quality tolerance class for unintended eccentricities, see 9.4.4.

(6) For shells fabricated in accordance with stricter fabrication tolerance quality classes than those defined by Class A at constructional details relevant to the fatigue verification and for which all the conditions for the application of the detail categories in EN 1993-1-9 are met, the factor $k_{f,imp}$ may be taken as 1,0.

Table10.1 — Values for the stress concentration factor $k_{f,imp}$ to compensate for unintended eccentricities not accounted for in the analysis

Fabrication tolerance quality class for unintended eccentricity	Consideration of unintended eccentricities in the analysis	
	Analysis without unintended eccentricities	Unintended eccentricities included in the analysis [1]
Class A	1,25	1,00
Class B	1,40	
Class C	1,70	

NOTE Planned and intended eccentricities that are part of the design are treated according to (2).

(7) The stress concentration factors $k_{f,imp}$ in (5) are applicable for misalignment imperfections at joints between shell courses of equal thickness where there is no intended eccentricity, such as is shown in Figure 9.6 a) and in Figure 9.7 b) and c).

(8) At joints between shell strakes of unequal thickness, such as those shown in Figure 9.6 b), reduced values of $k_{f,imp}$ may be adopted. The values given in Table 10.1 may be replaced by a reduced value that depends on the ratio of the plate thicknesses shown in Figure 9.6 b), found as

$$k_{f,imp} = 0,5\left\{3k_{o,ref} - 1 + \left(1 - k_{o.ref}\right)\left(\frac{t_{max}}{t_{min}}\right)\right\} \quad \text{but} \quad k_{f,imp} \geq 1,0 \tag{10.1}$$

in which $k_{o,ref}$ is the value of $k_{f,imp}$ given in Table 10.1.

(9) The largest surface stress may be conservatively used in place of the membrane stress normal to the orientation of the eccentricity in (5) and (6).

10.2.3 Multiaxial stress fields

(1) For construction details with linear geometric orientation, the stresses should be resolved into components transverse to and parallel to the axis of the detail.

NOTE 1 Careful assessment can be necessary adjacent to the termination of longitudinal stiffeners.

NOTE 2 Shell junctions are often geometrically complex, so that the evaluation of the stress at various key points can be necessary.

(2) Multiaxial fatigue loading should be verified in accordance with prEN 1993-1-9:2023, 9.4.

(3) Where different load cases with different numbers of occurrences and different stress ranges contribute to the fatigue evaluation, the damage caused by each case should be found separately by taking the stress field components transverse to and parallel to the axis of the constructional detail or notch and the damage contribution of each found using the cumulative linear damage model in prEN 1993-1-9:2023, Annex A.

NOTE For welded constructional details, the sign of the mean and maximum stress is not considered in the fatigue verification according to EN 1993-1-9. Modifications of the stress range can be used for non-welded or stress-relieved details where the largest absolute value of the stress in the design stress range is compressive, see prEN 1993-1-9:2023, 7.4.

(4) As an alternative to the procedure of (2) and (3), for constructional detail categories applicable for normal stresses as defined in EN 1993-1-9 the design values of stress range may be calculated using the following procedure:

— i. the largest absolute value of the principal stress resulting from fatigue actions and its orientation should be determined. If the principal stress orientation deviates by less than 45° from the stress direction indicated in the tables of constructional details in EN 1993-1-9, the calculated principal stress should be considered as the reference stress for the calculation of the stress range. Where the orientation does not satisfy this requirement, the stress component acting in the direction indicated in the tables of constructional details should be used as the reference stress in the following steps ii. and iii.

— ii. for other load cases that contribute to the considered fatigue loading event, the stresses should be resolved into the directions parallel to and normal to the reference stress defined in i;

— iii. the stress range associated for the evaluation of the fatigue loading event should be calculated using the stress components parallel to the reference stress defined in i;

— iv. in situations where various load cases produce maximum absolute values of principal stress of comparable amplitude but with differences in orientation that exceed 30°, the steps i. to iii. and the subsequent fatigue verification should be repeated for each appropriate direction of principal stress.

NOTE A fatigue loading event is defined as a period of time with a variation in magnitude and/or point of application of the fatigue action. It usually reoccurs a number of times.

10.2.4 Design values of resistance (fatigue strength)

(1) The fatigue resistance of the detail classes should be obtained from EN 1993-1-9 as follows.

— for the nominal and modified nominal stress approach, the tables in prEN 1993-1-9:2023, Clause 10 should be used.

— for the structural stress approach, the tables in prEN 1993-1-9:2023, Annex B should be used.

— for the effective notch stress approach, the tables in prEN 1993-1-9:2023, Annex C should be used.

(2) The partial factor for design resistance to fatigue γ_{Mf} should be taken from either EN 1993-1-9, application parts of EN 1993 or other applicable specification documents, as appropriate.

(3) The partial factor for fatigue loads and load effects γ_{Ff} should generally be chosen as γ_{Ff} =1,0 unless different values are given in application parts of EN 1993 or other applicable specification documents.

10.2.5 Fatigue verification

(1) The fatigue verification should be undertaken using in one of two alternative methods:

a) using equivalent constant stress ranges $\Delta\sigma_{e,2,Ed}$ as defined in prEN 1993-1-9:2023, 7.3.2 or 7.3.3 and applying the stressed-based verification format of prEN 1993-1-9:2023, Clause 9;

b) applying the verification format of linear cumulative damage models of prEN 1993-1-9:2023, Annex A (see also 10.1 (2)).

Annex A
(informative)

Membrane theory stresses in unstiffened shells

A.1 Use of this Annex

(1) This Informative Annex provides supplementary guidance on membrane theory stresses in unstiffened shells.

NOTE National choice on the application of this Informative Annex is given in the National Annex. If the National Annex contains no information on the application of this informative annex, it can be used.

A.2 Scope and field of application

(1) This Informative Annex covers rules on membrane theory stresses in unstiffened shells.

A.3 General

A.3.1 Action affects and resistances

(1) The action effects or resistances calculated using the formulae in this annex may be assumed to provide characteristic values of the action effect or resistance when characteristic values of the actions, geometric parameters and material properties are adopted.

A.3.2 Notation

(1) The notation used in this annex for the geometrical dimensions, stresses and loads follows 3.2. In addition, the following notation is used.

(2) Roman upper case letters:

F_x axial load applied to the cylinder;

F_z axial load applied to a cone;

M global bending moment applied to the complete cylinder (not to be confused with the moment per unit width in the shell wall m);

M_t global torque applied to the complete cylinder;

V global transverse shear applied to the complete cylinder.

(3) Roman lower case letters:

g unit weight of the material of the shell;

p_n distributed normal pressure;

p_x distributed axial traction on cylinder wall;

t shell wall thickness.

(4) Greek lower case letters:

β cone angle relative to the axis;

ϕ meridional slope angle;

σ_x axial membrane stress (= n_x/t);

σ_ϕ meridional membrane stress (= n_ϕ/t);

σ_θ circumferential membrane stress (= n_θ/t);

τ membrane shear stress (= $n_{x\theta}/t$).

A.3.3 Boundary conditions

(1) The boundary condition notations should be taken as detailed in 4.3 and 6.2.2.

(2) For these formulae to be strictly valid, the boundary conditions for cylinders should be taken as radially free at both ends, axially supported at one end, and rotationally free at both ends.

(3) For these formulae to be strictly valid for cones, the applied loading should match a membrane stress state in the shell and the boundary conditions should be taken as free to displace normal to the shell at both ends and meridionally supported at one end.

(4) For truncated cones, the boundary conditions should be understood to include components of loading transverse to the shell wall, so that the combined stress resultant introduced into the shell is solely in the direction of the shell meridian.

NOTE Where these formulae are used for axially stiffened shells, the provisions of EN 1993-4-1 can be used to obtain the differences between the stresses in the shell and the stiffener.

A.3.4 Sign convention

(1) The sign convention for stresses σ should be taken everywhere as tension positive, though some of the Figures illustrate cases in which the external load is applied in the opposite sense.

A.4 Cylindrical shells

Table A.1 — Loads inducing axial stresses

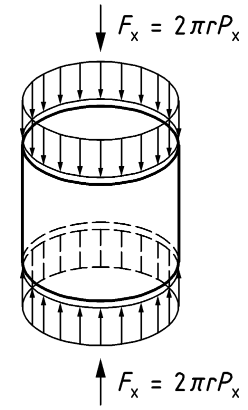
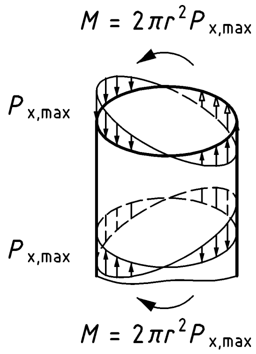
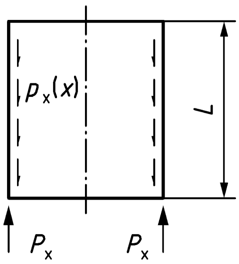
Uniform axial load		$\sigma_x = -\dfrac{F_x}{2\pi rt}$
Axial stresses from global bending		$\sigma_x = \pm\dfrac{M}{\pi r^2 t}$
Axial stresses from frictional load		$\sigma_x = -\dfrac{\int_0^L p_x \cdot dx}{t}$

Table A.2 — Loads inducing circumferential stresses

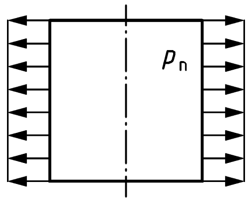
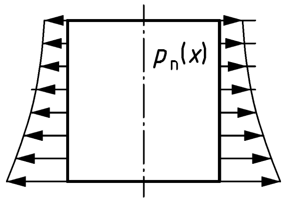
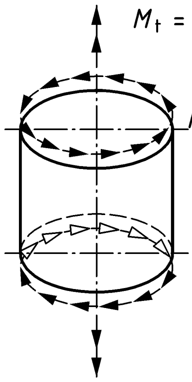
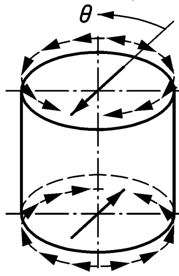
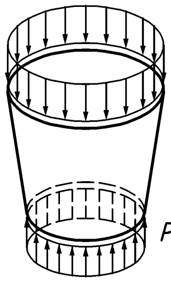
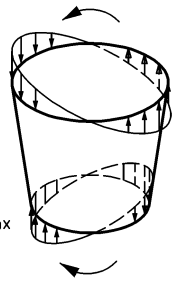
Uniform internal pressure		$\sigma_\theta = p_n \cdot \dfrac{r}{t}$
Varying internal pressure		$\sigma_\theta(x) = p_n(x) \cdot \dfrac{r}{t}$

Table A.3 — Loads inducing membrane shear

Uniform shear from torsion		$\tau = \frac{M_t}{2\pi r^2 t}$
Sinusoidal shear from transverse force		$\tau_{max} = \pm \frac{V}{\pi r t}$ $P_\theta = P_{\theta,max} \sin\theta$

A.5 Conical shells

Table A.4 — Loads inducing meridional stresses

Meridional stresses from axial load		$\sigma_\phi = -\frac{F_z}{2\pi r t \cdot \cos\beta}$ $\sigma_\theta = 0$
Meridional stresses from global bending		$\sigma_{\phi,max} = \pm \frac{M}{\pi r^2 t \cdot \cos\beta}$ $\sigma_\theta = 0$

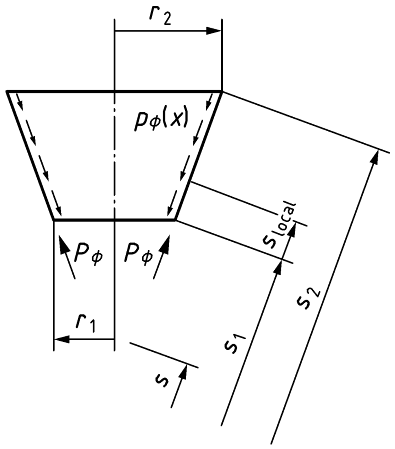
Meridional stresses from frictional load		$\sigma_{\phi 1} = -\frac{1}{s_1 t}\int_{s_1}^{s_2} p_\phi s \cdot ds$ $\sigma_\theta = 0$
	$s_1 = r_1/\sin\beta$ $s_2 = r_2/\sin\beta$	

Table A.5 — Loads inducing circumferential stresses

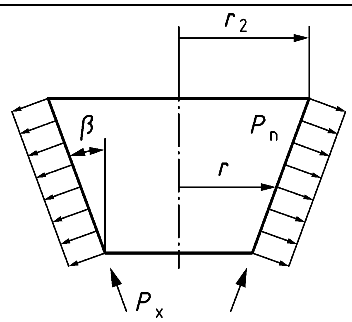
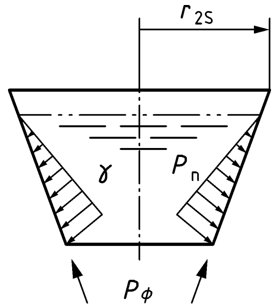
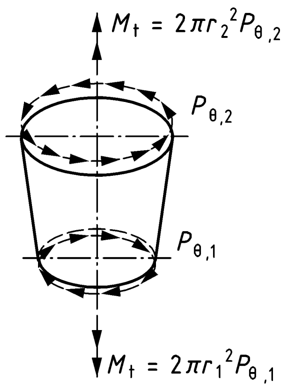
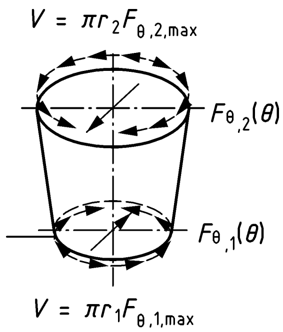
Uniform internal pressure		$\sigma_\phi = -p_n \frac{r}{2t \cdot \cos\beta}\left[\left(\frac{r_2}{r}\right)^2 - 1\right]$ $\sigma_\theta = p_n \frac{r}{t \cdot \cos\beta}$
Linearly varying internal pressure		$\sigma_\phi = -\frac{\gamma r}{t \cdot \sin\beta}\left\{\frac{r_{2s}}{6}\left[\left(\frac{r_{2s}}{r}\right)^2 - 3\right] + \frac{r}{3}\right\}$ $\sigma_\theta = +\frac{\gamma r}{t \cdot \sin\beta}(r_{2s} - r)$ r_{2S} is the radius at the fluid surface

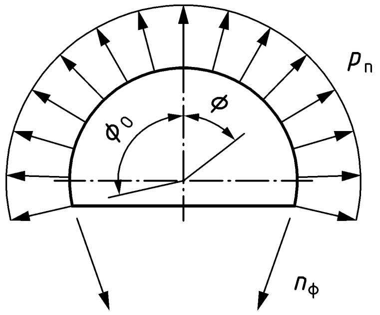
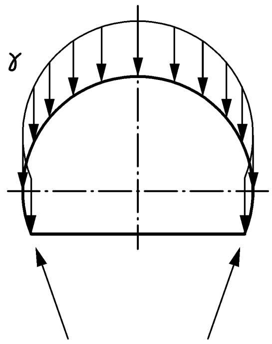
Table A.6 — Loads inducing membrane shear stresses

Non-uniform shear induced by torsion (note that the shear stress varies quadratically down the cone)		$\tau = \frac{M_t}{2\pi r^2 t}$

Sinusoidal shear from transverse force (note that the shear stress varies quadratically down the cone) Sinusoidal shear from transverse force		$\tau_{\max} = \pm \frac{V}{\pi r t}$
	$F_{\theta,i} = F_{\theta,i,max} \sin \theta$	

A.6 Spherical shells

Table A.7 — Loads inducing membrane stresses

Uniform internal pressure		$\sigma_\phi = \frac{p_n r_s}{2t}$ $\sigma_\theta = -\frac{\gamma r_s}{t}\left(\cos\phi - \frac{1}{1+\cos\phi}\right)$ $\sigma_\theta = \frac{p_n r_s}{2t}$
Uniform self-weight		$\sigma_\phi = -\frac{\gamma r_s}{t}\left(\frac{1}{1+\cos\phi}\right)$

where

r_s is the radius of the sphere;

γ is the unit weight of the material of construction;

ϕ is the local meridional slope of the shell;

σ_ϕ is the meridional membrane stress;

σ_θ is the circumferential membrane stress.

Annex B
(informative)

Formulae for plastic reference resistances of unstiffened shells and circular plates

B.1 Use of this Annex

(1) This Informative Annex provides formulae for plastic reference resistances of unstiffened shells and circular plates.

NOTE National choice on the application of this Informative Annex is given in the National Annex. If the National Annex contains no information on the application of this informative annex, it can be used.

B.2 Scope and field of application

(1) This Informative Annex provides formulae for plastic reference resistances of unstiffened shells and circular plates.

B.3 General

B.3.1 Resistances

(1) The resistances calculated using the formulae in this annex may be assumed to provide characteristic values of the reference plastic resistance when characteristic values of the geometric parameters and material properties are adopted.

(2) Where a radial line ring load is defined, the same plastic resistance is found for both inward and outward loads.

(3) Where an axial load is defined, the formulae are valid for both tensile and compressive loads.

NOTE Where ring stiffeners are present, the information in this annex relates to the unstiffened segments between the stiffeners.

B.3.2 Notation

(1) The notation used in this annex for the geometrical dimensions, stresses and loads follows 3.2. In addition, the following notation is used.

(2) Roman upper case letters:

A_r cross-sectional area of a ring;

P_R characteristic value of small deflection theory plastic mechanism resistance in terms of forces (MNA resistance).

(3) Roman lower case letters:

b thickness of a ring;

ℓ_o effective length of shell which acts with a ring $\ell_o = 0{,}975\sqrt{rt}$;

p_R characteristic value of small deflection theory plastic mechanism resistance in terms of pressure (MNA resistance).

r radius of the cylinder;

s_{eq} dimensionless von Mises equivalent stress parameter;

s_m dimensionless combined stress parameter;

s_x dimensionless axial stress parameter;

s_θ dimensionless circumferential stress parameter.

(4) Subscripts:

r relating to a ring.

B.3.3 Boundary conditions

(1) The boundary condition notations are detailed in 6.2.2.

(2) The term "clamped" should be taken to refer to BC1r and the term "pinned" to refer to BC2f.

(3) The reference length of the cylinder in plastic evaluations is given by $\ell_o = 0,975\sqrt{rt}$

B.4 Uniform unstiffened cylindrical shells

B.4.1 Radial ring line load

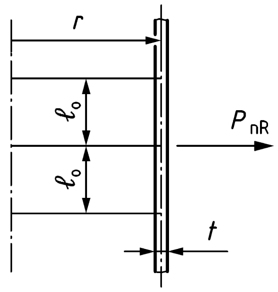


Figure B.1 — Radial ring line load on a cylinder

The plastic resistance P_{nR} (force per unit circumference) is given by:

$$P_{nR} = 2\ell_o \left(\frac{t}{r}\right) f_y$$

B.4.2 Radial outward ring line load and axial tension

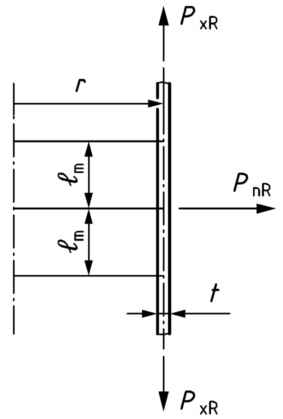


Figure B.2 — Radial ring line load with axial tension on a cylinder

The relative magnitude of the axial force is given by

$s_x = \dfrac{P_x}{f_y t}$ with relevance in the range $-1 \le s_x \le +1$

The parameter A should be found using

where $P_n > 0$ (outward) then: $A = + s_x - 1,50$

where $P_n < 0$ (inward) then: $A = - s_x - 1,50$

The von Mises parameter is given by

$$s_{eq} = A + \sqrt{A^2 + A\left(1 - s_x^2\right)}$$

The effective length ℓ_m is given by $\ell_m = s_{eq}\, \ell_o$

Where $s_x = 0$, this expression is not relevant and the provisions of B.4.1 should be used.

The plastic resistance P_{nR} (force per unit circumference) is given by:

$$\frac{P_{nR}}{2\ell_m} = f_y \frac{t}{r}$$

B.4.3 Radial ring line load, internal pressure and axial load

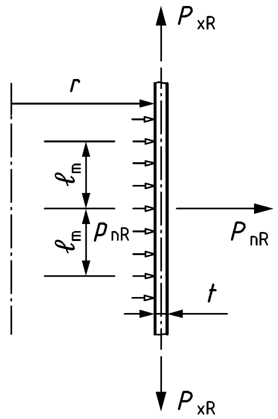


Figure B.3 — Radial ring line load with internal pressure and axial load on a cylinder

The relative magnitude of the axial force is given by

$s_x = \dfrac{P_x}{f_y t}$ with relevance for the range $-1 \leq s_x \leq +1$

The relative magnitude of the circumferential stress is given by $s_\theta = \left(\dfrac{p_n}{f_y}\right)\left(\dfrac{r}{t}\right)$ with $-1 \leq s_\theta \leq +1$

The von Mises parameter is given by $s_{eq} = \sqrt{s_\theta^2 + s_x^2 - s_x s_\theta}$

Table B.1 — Determination of the length measure ℓ_m

Outward directed ring load $P_n > 0$		Inward directed ring load $P_n < 0$	
Condition	Formulae	Condition	Formulae
$s_{eq} < 1{,}00$ and $s_\theta \leq 0{,}975$	$A = + s_x - 2s_\theta - 1{,}50$ $s_m = A + \sqrt{A^2 + A\left(1 - s_{eq}^2\right)}$ $\ell_m = \ell_o \left(\dfrac{s_m}{1 - s_\theta}\right)$	$s_{eq} < 1{,}00$ and $s_\theta \geq -0{,}975$	$A = - s_x + 2s_\theta - 1{,}50$ $s_m = A + \sqrt{A^2 + A\left(1 - s_{eq}^2\right)}$ $\ell_m = \ell_o \left(\dfrac{s_m}{1 + s_\theta}\right)$
$s_{eq} = 1{,}00$ or $s_\theta > 0{,}975$	$\ell_m = 0{,}0$	$s_{eq} = 1{,}00$ or $s_\theta < -0{,}975$	$\ell_m = 0{,}0$

The plastic resistance, for any combination of P_n and p_n, is given by:

$$\frac{P_{nR}}{2\ell_m} + p_n = f_y \left(\frac{t}{r}\right)$$

in which P_n and p_n are in positive outwards sign convention.

B.5 Cylindrical shells with local ring stiffeners

B.5.1 Radial line ring load alone

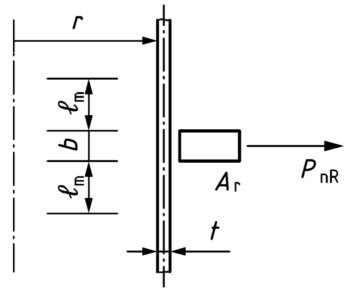


Figure B.4 — Radial line ring load on a ring stiffener attached to a cylinder

The plastic resistance P_{nR} (force per unit circumference) is given by:

$$P_{nR} = f_y \left(\frac{A_r + (b + 2\ell_m)t}{r} \right)$$

in which $\ell_m = \ell_o$

B.5.2 Radial line ring load with axial load

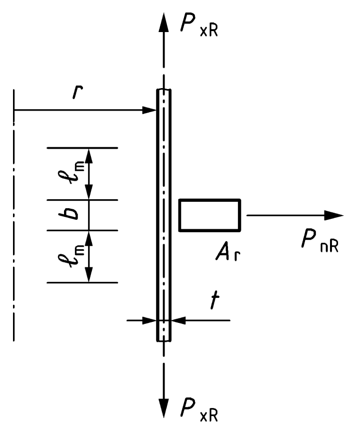


Figure B.5 — Radial line ring load on a ring stiffener attached to a cylinder with axial load

The relative magnitude of the axial force is given by

$s_x = \dfrac{P_x}{f_y t}$ with relevance for the range $-1 \le s_x \le +1$

The parameter A should be found using

Where $P_n > 0$ then: $A = + s_x - 1{,}50$

Where $P_n < 0$ then: $A = - s_x - 1{,}50$

The von Mises parameter is given by

$$s_m = A + \sqrt{A^2 + A\left(1 - s_x^2\right)}$$

Where $s_x \neq 0$ then: $\ell_m = s_m \ell_o$

The plastic resistance P_{nR} (force per unit circumference) is given by:

$$P_{nR} = f_y \left(\frac{A_r + (b + 2\ell_m)t}{r} \right)$$

B.5.3 Radial line ring load, internal pressure and axial load

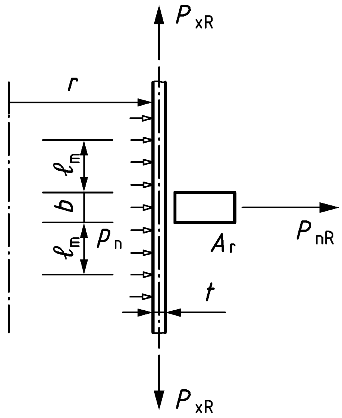


Figure B.6 — Radial line ring load on a ring stiffener attached to a cylinder with axial load and internal pressure

The relative magnitude of the axial force is given by

$s_x = \dfrac{P_x}{f_y t}$ with relevance for the range $-1 \le s_x \le +1$

The relative magnitude of the circumferential force is given by

$s_\theta = \dfrac{p_n}{f_y} \cdot \dfrac{r}{t}$ with relevance for the range $-1 \le s_\theta \le +1$

The von Mises parameter should be found as $s_{eq} = \sqrt{s_\theta^2 + s_x^2 - s_x s_\theta}$

Table B.2 — Parameters in the plastic resistance evaluation

Outward directed ring load $P_n > 0$		**Inward directed ring load $P_n < 0$**	
Condition	Formulae	Condition	Formulae
$s_{eq} < 1{,}00$ and $s_\theta \le 0{,}975$	$A = + s_x - 2s_\theta - 1{,}50$ $s_m = A + \sqrt{A^2 + A\left(1 - s_{eq}^2\right)}$ $\ell_m = \ell_o \left(\dfrac{s_m}{1 - s_\theta} \right)$	$s_{eq} < 1{,}00$ and $s_\theta \ge -0{,}975$	$A = - s_x + 2s_\theta - 1{,}50$ $s_m = A + \sqrt{A^2 + A\left(1 - s_{eq}^2\right)}$ $\ell_m = \ell_o \left(\dfrac{s_m}{1 + s_\theta} \right)$
$s_{eq} = 1{,}00$ or $s_\theta > 0{,}975$	$\ell_m = 0{,}0$	$s_{eq} = 1{,}00$ or $s_\theta < -0{,}975$	$\ell_m = 0{,}0$

with P_{nR} and p_n in positive outward sign convention, the plastic resistance is given by :

$$P_{nR} + p_n\,(b + 2\ell_m) = f_y \left(\frac{A_r + (b + 2\ell_m)t}{r} \right)$$

B.6 Junctions between conical and cylindrical shells

B.6.1 Meridional forces alone (simplified)

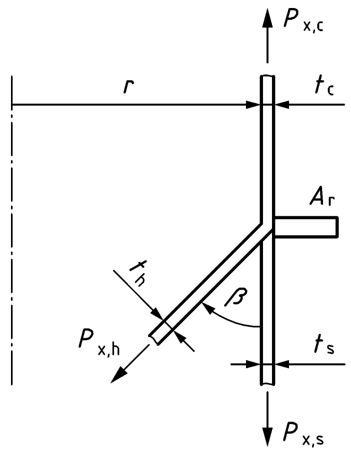


Figure B.7 — Cone-cylinder junction under only meridional forces

The following description is valid within the ranges:

$t_c^2 \le t_s^2 + t_h^2$ with $|P_{x,s}| << t_s f_y$, $|P_{x,h}| << t_h f_y$ and $|P_{x,c}| << t_c f_y$`

The relative contributions of the different shell segments is found using:

$\eta = \sqrt{\frac{t_c^2}{t_s^2 + t_h^2}}$ and $\psi_s = \psi_h = 0{,}7 + 0{,}6\eta^2 - 0{,}3\eta^3$

For the cylinder $\ell_{oc} = 0{,}975\sqrt{rt_c}$

For the skirt $\ell_{os} = 0{,}975\ \psi_s\sqrt{rt_s}$

For the conical segment $\ell_{oh} = 0{,}975\ \psi_h\sqrt{\frac{rt_h}{\cos\beta}}$

The plastic resistance, defined in terms of the meridional membrane force per unit circumference at the top of the cone P_{xhR}, is given by:

$P_{xhR}\, r \sin\beta = f_y\, (A_r + \ell_{oc}t_c + \ell_{os}t_s + \ell_{oh}t_h)$

B.6.2 Internal pressure and meridional forces

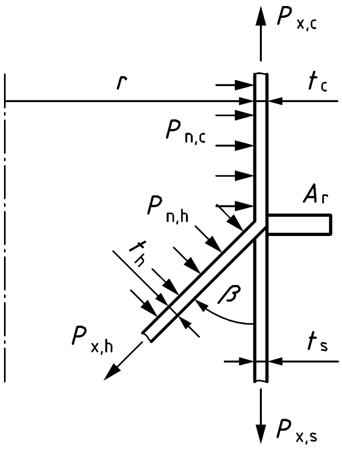


Figure B.8 — Cone-cylinder junction with internal pressure and meridional forces

The relative magnitudes of the forces in each of the three segments c, s and h are defined by:

$s_{xc} = \dfrac{P_{x,c}}{f_y t_c}$ and $s_{xs} = \dfrac{P_{x,s}}{f_y t_s}$ and $s_{xh} = \dfrac{P_{x,h}}{f_y t_h}$

$s_{\theta c} = \dfrac{p_{n,c}}{f_y} \cdot \dfrac{r}{t_c}$ and $s_{\theta s} = 0$ and $s_{\theta h} = \dfrac{p_{n,h}}{f_y} \cdot \dfrac{r}{t_h \cdot \cos\beta}$

for i = c, s, h in turn, the von Mises equivalent measure should be found using

$$s_{eqi} = \sqrt{s_{\theta i}^2 + s_{xi}^2 - s_{xi}s_{\theta i}}$$

in which the subscripts c, s and h refer to the cylinder, skirt and hopper respectively.

The following description is valid within the ranges, applied in turn to each shell segment:

$-1 \le s_{xi} \le +1 \qquad -1 \le s_{\theta i} \le +1$

The equivalent thickness of the segments above and below the junction should be found using the parameters ψ_c , ψ_s and ψ_h as defined in Table B.3.

Table B.3 — Equivalent thickness evaluation

Lower plate group thicker $t_c^2 \le t_s^2 + t_h^2$	Upper plate group thicker $t_c^2 > t_s^2 + t_h^2$
$\eta = \sqrt{\dfrac{t_c^2}{t_s^2 + t_h^2}}$ $\psi_c = 1{,}0$ $\psi_s = \psi_h = 0{,}7 + 0{,}6\eta^2 - 0{,}3\eta^3$	$\eta = \sqrt{\dfrac{t_s^2 + t_h^2}{t_c^2}}$ $\psi_c = 0{,}7 + 0{,}6\eta^2 - 0{,}3\eta^3$ $\psi_s = \psi_h = 1{,}0$

For the cylindrical segments the effective length is given by $\ell_{oi} = 0{,}975\ \psi_i \sqrt{rt_i}$

For the conical segment the effective length is given by $\ell_{oh} = 0{,}975\ \psi_h \sqrt{\dfrac{rt_i}{\cos\beta}}$

The values of the lengths of shell contributing to the plastic mechanism $\boldsymbol{\ell}_{mc}$, $\boldsymbol{\ell}_{ms}$ and $\boldsymbol{\ell}_{mh}$ should be determined using Table B.4.

Table B.4 — Parameters for plastic resistance evaluation

For each shell segment ***i*** separately	
Condition	Formulae
$s_{eqi} < 1{,}00$ and $s_{\theta i} \geq -0{,}975$	$A_i = -s_{xi} + 2s_{\theta i} - 1{,}50$ $s_{mi} = A_i + \sqrt{A_i^2 + A\left(1 - s_{ei}^2\right)}$ $\ell_{mi} = \ell_{oi}\left(\frac{s_{mi}}{1 + s_{\theta i}}\right)$
$s_{eqi} = 1{,}00$	$\ell_{mi} = 0{,}0$
$s_{\theta i} < -0{,}975$	$\ell_{mi} = 0{,}0$

The plastic resistance of the complete junction, defined in terms of the meridional membrane force per unit circumference at the top of the cone P_{xhR}, is given by:

$$P_{xhR}\, r \sin\beta = f_y\, (A_r + \boldsymbol{\ell}_{mc} t_c + \boldsymbol{\ell}_{ms} t_s + \boldsymbol{\ell}_{mh} t_h) + r\, (p_{nc} \boldsymbol{\ell}_{mc} + p_{nh} \boldsymbol{\ell}_{mh} \cos\beta)$$

B.7 Circular plates with axisymmetric boundary conditions

B.7.1 Uniform transverse pressure with simply supported boundary

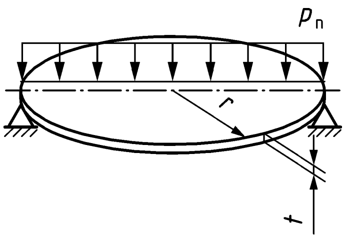


Figure B.9 — Simply supported circular plate under uniform transverse pressure

The plastic resistance pressure $p_{n,R}$ is given by

$$p_{n,R} = 1{,}625\left(\frac{t}{r}\right)^2 f_y$$

B.7.2 Central circular patch of transverse pressure with simply supported boundary

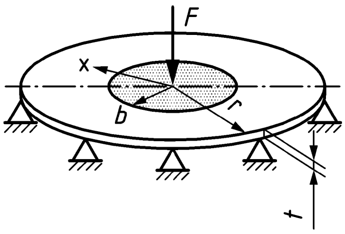


Figure B.10 — Simply supported circular plate under central circular patch of transverse pressure

For uniform pressure p_n on circular patch of radius b with total load $F = p_n\, \pi\, b^2$, the plastic resistance F_R is given by

$$F_R = K\frac{\pi}{2}t^2 f_y$$

where K is taken as the lower of the two values:

$$K = 1{,}0 + 1{,}10\left(\frac{b}{r}\right) + 1{,}15\left(\frac{b}{r}\right)^4 \quad \text{or} \quad K = \frac{1}{\sqrt{3}}\cdot\frac{b}{t}$$

B.7.3 Uniform transverse pressure with clamped boundary

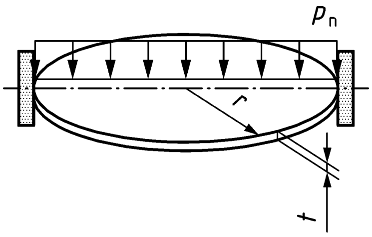


Figure B.11 — Clamped supported circular plate under uniform transverse pressure

The plastic resistance pressure $p_{n,R}$ is given by

$$p_{n,R} = 3{,}125\left(\frac{t}{r}\right)^2 f_y$$

B.7.4 Central circular patch of transverse pressure with clamped boundary

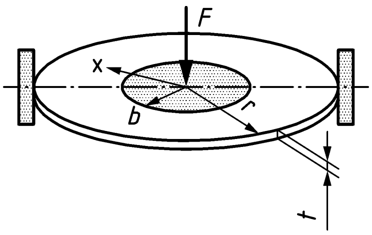


Figure B.12 — Clamped supported circular plate under central circular patch of transverse pressure

For uniform pressure p_n on circular patch of radius b with total load $F = p_n \pi b^2$, the plastic resistance F_R is given by

$$F_R = K\frac{\pi}{2}t^2 f_y$$

where K is the lesser of

$$K = 1{,}40 + 2{,}85\frac{b}{r} + 2{,}0\left(\frac{b}{r}\right)^4 \quad \text{and} \quad K = \frac{1}{\sqrt{3}}\cdot\frac{b}{t}$$

Annex C
(informative)

Formulae for linear elastic membrane and bending stresses in unstiffened cylindrical shells and circular plates

C.1 Use of this Annex

(1) This Informative Annex provides formulae for linear elastic membrane and bending stresses in unstiffened cylindrical shells and circular plates.

NOTE National choice on the application of this Informative Annex is given in the National Annex. If the National Annex contains no information on the application of this informative annex, it can be used.

C.2 Scope and field of application

(1) This Informative Annex provides formulae for linear elastic membrane and bending stresses in unstiffened cylindrical shells and circular plates.

C.3 General

C.3.1 Action effects

(1) The action effects calculated using the formulae in this annex may be assumed to provide characteristic values of the action effect when characteristic values of the actions, geometric parameters and material properties are adopted.

(2) For fatigue design (LS4), the ratio between the stresses calculated using this Annex and the stresses calculated using membrane theory (e.g. Annex A) can be used in two ways

a) to calculate modified nominal stress ranges (prEN 1993-1-9:2023, 7.3.1 (2));

b) to determine stress concentration factors k_f, (prEN 1993-1-9:2023, 7.3.3 and prEN 1993-1-9:2023, Annex D).

(3) The modified nominal stresses for fatigue design calculated in this manner may not yet cover the effects of misalignment imperfections that exceed the minimum tolerance defined in EN 1993-1-9 and consequently may require further modification, see 10.2.2 (3) and (4).

C.3.2 Notation

(1) The notation used in this annex for the geometrical dimensions, stresses and loads follows 3.2. In addition, the following notation is used.

(2) Roman characters:

b radius at which local load on plate terminates;

r outside radius of circular plate;

x axial coordinate on cylinder or radial coordinate on circular plate.

(3) Greek symbols:

$\sigma_{eq,m}$ von Mises equivalent stress based on the membrane stress components alone (see 7.2.1);

$\sigma_{eq,s}$ von Mises equivalent stress based on surface stresses (see 7.2.1);

σ_{MT} reference stress derived from membrane theory;

σ_{bx} meridional bending stress;

$\sigma_{b\theta}$ circumferential bending stress;

σ_{sx} meridional surface stress;

$\sigma_{s\theta}$ circumferential surface stress;

τ_{xn} transverse shear stress associated with meridional bending.

(4) Subscripts:

n normal;

r relating to a ring;

y first yield value.

C.3.3 Boundary conditions

(1) The boundary condition notations should be taken as detailed in 6.2.2.

(2) The term "clamped" refers to BC1r.

(3) The term "pinned" refers to BC2f.

C.4 Clamped base cylindrical shells

C.4.1 Uniform internal pressure

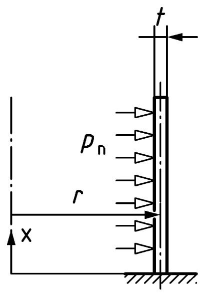


Figure C.1 — Clamped cylinder under uniform internal pressure

The reference membrane theory circumferential stress is $\sigma_{MT\theta} = p_n \frac{r}{t}$

With BC1r, the maximum surface stress and von Mises equivalent stress are given in Table C.1.

Table C.1 — Maximum stress values

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$\pm 1{,}816\ \sigma_{MT\theta}$	$+1{,}080\ \sigma_{MT\theta}$	$1{,}169\sqrt{t/r}\ \sigma_{MT\theta}$	$1{,}614\ \sigma_{MT\theta}$	$1{,}043\ \sigma_{MT\theta}$

C.4.2 Axial loading

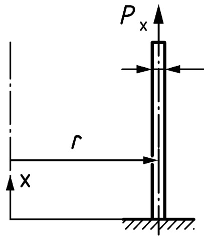


Figure C.2 — Clamped cylinder under axial load

The reference membrane theory axial stress is given by $\sigma_{MTx} = \frac{P_x}{t}$

With BC1r, the maximum surface stress and von Mises equivalent stress are given in Table C.2.

Table C.2 — Maximum stress values

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$1{,}545\ \sigma_{MTx}$	$+0{,}455\ \sigma_{MTx}$	$0{,}351\sqrt{t/r}\ \sigma_{MTx}$	$1{,}373\ \sigma_{MTx}$	$1{,}000\ \sigma_{MTx}$

C.4.3 Uniform internal pressure with axial loading

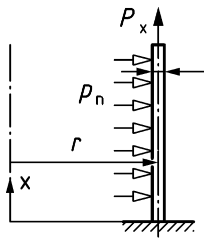


Figure C.3 — Clamped cylinder under axial load with internal pressure

The reference membrane theory circumferential stress is $\sigma_{MT\theta} = p_n \frac{r}{t}$

The reference membrane theory axial stress is $\sigma_{MTx} = \frac{P_x}{t}$

With BC1r, the maximum surface stress and von Mises equivalent stress are given by

$$\text{Maximum } \sigma_{eq,m} = \sigma_{MT\theta} \sqrt{1 - \left(\frac{\sigma_{MTx}}{\sigma_{MT\theta}}\right) + \left(\frac{\sigma_{MTx}}{\sigma_{MT\theta}}\right)^2}$$

$$\text{Maximum } \sigma_{eq,m} = k\ \sigma_{MT\theta}$$

The maximum stress values depend on the ratio of axial to circumferential stress $\left(\dfrac{\sigma_{MTx}}{\sigma_{MT\theta}}\right)$ and are given in Table C.3.

Table C.3 — Maximum stress values

$\left(\dfrac{\sigma_{MTx}}{\sigma_{MT\theta}}\right)$	−2,0	0	0	2,0
	Outer surface controls		Inner surface controls	
k	4,360	1,614	1,614	2,423

Linear interpolation in $\left(\dfrac{\sigma_{MTx}}{\sigma_{MT\theta}}\right)$ may be used between values where the same surface controls.

C.4.4 Hydrostatic internal pressure

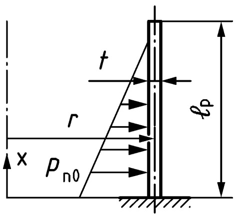


Figure C.4 — Clamped cylinder under hydrostatic internal pressure

The reference membrane theory circumferential stress is $\sigma_{MT\theta} = p_{n0}\dfrac{r}{t}$

For BC1r the maximum surface stress and von Mises equivalent stress are given in Tables C.4 and C.5.

Table C.4 — Maximum stress notation

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$k_x\ \sigma_{MT\theta}$	$k_\theta\ \sigma_{MT\theta}$	$k_\tau\ \sqrt{t/r}\ \ \sigma_{MT\theta}$	$k_{eq,s}\ \sigma_{MT\theta}$	$k_{eq,m}\ \sigma_{MT\theta}$

Table C.5 — Maximum stress values for different lengths ℓ_p

$\left(\dfrac{\sqrt{rt}}{\ell_p}\right)$	k_x	k_θ	$k_{x\theta}$	$k_{eq,s}$	$k_{eq,m}$
0	1,816	1,080	1,169	1,614	1,043
0,2	1,533	0,733	1,076	1,363	0,647

Linear interpolation in $\left(\dfrac{\sqrt{rt}}{\ell_p}\right)$ may be used between the defined values.

C.4.5 Radial outward base displacement

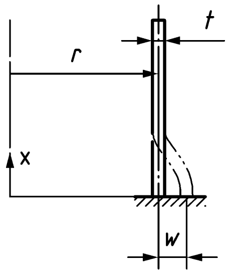


Figure C.5 — Clamped cylinder with radial outward base displacement

The reference membrane theory circumferential stress is $\sigma_{MT\theta} = \frac{wE}{r}$

With BC1r, the maximum surface stress and von Mises equivalent stress are given in Table C.6.

Table C.6 — Maximum stress values

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
1,816 $\sigma_{MT\theta}$	1,545 $\sigma_{MT\theta}$	1,169 $\sqrt{t/r}$ $\sigma_{MT\theta}$	2,081 $\sigma_{MT\theta}$	1,000 $\sigma_{MT\theta}$

C.4.6 Uniform temperature rise

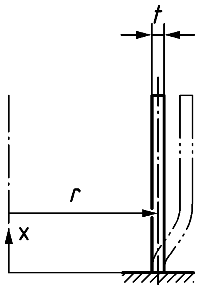


Figure C.6 — Clamped cylinder with uniform temperature rise

The reference membrane theory circumferential stress is $\sigma_{MT\theta} = \alpha\, E\, T$

The reference membrane outward displacement is $w = \alpha\, r\, T$

With BC1r, the maximum surface stress and von Mises equivalent stress are given in Table C.7.

Table C.7 — Maximum stress values

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
1,816 $\sigma_{MT\theta}$	1,545 $\sigma_{MT\theta}$	1,169 $\sqrt{t/r}$ $\sigma_{MT\theta}$	2,081 $\sigma_{MT\theta}$	1,000 $\sigma_{MT\theta}$

C.5 Pinned base cylindrical shells

C.5.1 Uniform internal pressure

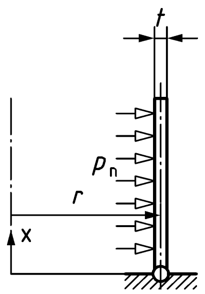


Figure C.7 — Pinned cylinder under uniform internal pressure

The reference membrane theory circumferential stress is $\sigma_{MT\theta} = p_n \dfrac{r}{t}$

With BC1f, the maximum surface stress and von Mises equivalent stress are given in Table C.8.

Table C.8 — Maximum stress values

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
±0,585 $\sigma_{MT\theta}$	+1,125 $\sigma_{MT\theta}$	0,583 $\sqrt{t/r}$ $\sigma_{MT\theta}$	1,126 $\sigma_{MT\theta}$	1,067 $\sigma_{MT\theta}$

C.5.2 Axial loading

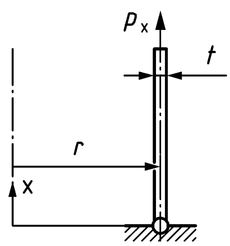


Figure C.8 — Pinned cylinder under axial load

The reference membrane theory circumferential stress is $\sigma_{MTx} = \dfrac{P_x}{t}$

With BC1f, the maximum surface stress and von Mises equivalent stress are given in Table C.9.

Table C.9 — Maximum stress values

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
+1,176 σ_{MTx}	+0,300 σ_{MTx}	0,175 $\sqrt{t/r}$ σ_{MTx}	1,118 σ_{MTx}	1,010 σ_{MTx}

C.5.3 Uniform internal pressure with axial loading

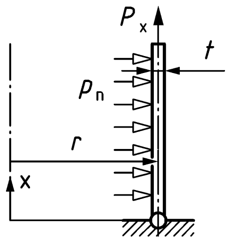


Figure C.9 — Pinned cylinder under axial load with internal pressure

The reference membrane theory stress are $\sigma_{\mathrm{MT\theta}} = p_{\mathrm{n}} \frac{r}{t}$ and $\sigma_{\mathrm{MTx}} = \frac{P_{\mathrm{x}}}{t}$

With BC1f, the maximum von Mises equivalent membrane stress $\sigma_{eq,m}$ is given by

Maximum $\sigma_{\mathrm{eq,m}} = \sigma_{\mathrm{MT\theta}} \sqrt{1 - \left(\frac{\sigma_{\mathrm{MTx}}}{\sigma_{\mathrm{MT\theta}}}\right) + \left(\frac{\sigma_{\mathrm{MTx}}}{\sigma_{\mathrm{MT\theta}}}\right)^2}$

The maximum von Mises equivalent surface stress $\sigma_{eq,s}$ is given by the values of k in Table C.10 with Maximum $\sigma_{eq,s} = k\, \sigma_{\mathrm{MT\theta}}$

Table C.10 — Maximum stress values for different axial to internal pressure ratios

$\left(\frac{\sigma_{MTx}}{\sigma_{MT\theta}}\right)$	−2,0	−1,0	−0,5	0,0	0,25	0,50	1,00	2,0
k	3,146	3,075	1,568	1,126	0,971	0,991	1,240	1,943

Linear interpolation in $\left(\frac{\sigma_{MTx}}{\sigma_{MT\theta}}\right)$ may be used between the defined values.

C.5.4 Hydrostatic internal pressure

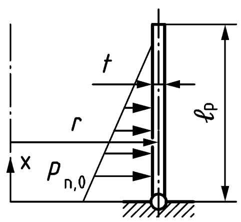


Figure C.10 — Pinned cylinder under hydrostatic internal pressure

The reference membrane theory circumferential stress is $\sigma_{MT\theta} = p_{n0} \frac{r}{t}$

With BC1f, the maximum surface stress and von Mises equivalent stress are given in Tables C.11 and C.12.

Table C.11 — Maximum stress notation

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$k_x\, \sigma_{MT\theta}$	$k_\theta\, \sigma_{MT\theta}$	$k_\tau \sqrt{t/r}\, \sigma_{MT\theta}$	$k_{eq,s}\, \sigma_{MT\theta}$	$k_{eq,m}\, \sigma_{MT\theta}$

Table C.12 — Maximum stress values for different lengths ℓ_p

$\left(\frac{\sqrt{rt}}{\ell_p}\right)$	k_x	k_θ	k_τ	$k_{eq,s}$	$k_{eq,m}$
0	0,585	1,125	0,583	1,126	1,067
0,2	0,585	0,873	0,583	0,919	0,759

Linear interpolation in $\left(\frac{\sqrt{rt}}{\ell_p}\right)$ may be used for different values of ℓ_p.

C.5.5 Radial outward base displacement

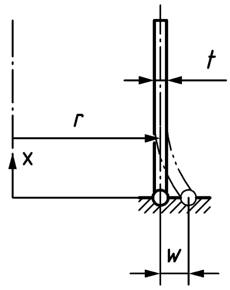


Figure C.11 — Pinned cylinder with radial outward base displacement

The reference membrane theory circumferential stress is $\sigma_{MT\theta} = \frac{wE}{r}$

With BC1f, the maximum surface stress σ_s and von Mises equivalent stress σ_{eq} are given in Table C.13.

Table C.13 — Maximum stress values

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$\pm 0,585\ \sigma_{MT\theta}$	$1,000\ \sigma_{MT\theta}$	$0,583\sqrt{t/r}\ \sigma_{MT\theta}$	$1,000\ \sigma_{MT\theta}$	$1,000\ \sigma_{MT\theta}$

C.5.6 Uniform temperature rise

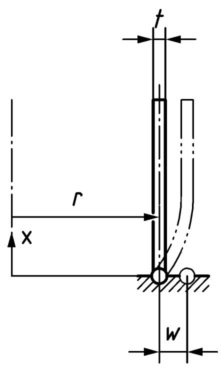


Figure C.12 — Pinned cylinder under uniform temperature rise *T*

The reference membrane theory circumferential stress is $\sigma_{MT\theta} = \alpha\, E\, T$

The membrane outward displacement is given by $w = \alpha\, r\, T$

With BC1f, the maximum surface stress σ_s and von Mises equivalent stress σ_{eq} are given in Table C.14.

Table C.14 — Maximum stress values

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
±0,585 $\sigma_{MT\theta}$	1,000 $\sigma_{MT\theta}$	0,583 $\sqrt{t/r}$ $\sigma_{MT\theta}$	1,000 $\sigma_{MT\theta}$	1,000 $\sigma_{MT\theta}$

C.5.7 Boundary rotation

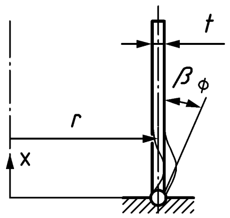


Figure C.13 — Pinned cylinder with base rotation β_ϕ

The reference membrane theory circumferential stress is $\sigma_{MT\theta} = E\sqrt{\frac{t}{r}} \cdot \beta_\phi$

With BC1f, the maximum surface stress σ_s and von Mises equivalent stress σ_{eq} are given in Table C.15.

Table C.15 — Maximum stress values

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
±1,413 $\sigma_{MT\theta}$	0,470 $\sigma_{MT\theta}$	0,454 $\sqrt{t/r}$ $\sigma_{MT\theta}$	1,255 $\sigma_{MT\theta}$	0,251 $\sigma_{MT\theta}$

C.6 Internal conditions in cylindrical shells

C.6.1 Step change of internal pressure

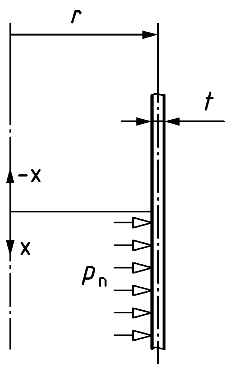


Figure C.14 — Abrupt step change in internal pressure

The reference membrane theory circumferential stress is $\sigma_{MT\theta} = p_n \frac{r}{t}$

The maximum surface stress σ_s and von Mises equivalent stress σ_{eq} are given in Table C.16.

Table C.16 — Maximum stress values

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$\pm 0{,}293\ \sigma_{MT\theta}$	$1{,}062\ \sigma_{MT\theta}$	$0{,}467\sqrt{t/r}\ \sigma_{MT\theta}$	$1{,}056\ \sigma_{MT\theta}$	$1{,}033\ \sigma_{MT\theta}$

C.6.2 Hydrostatic internal pressure termination

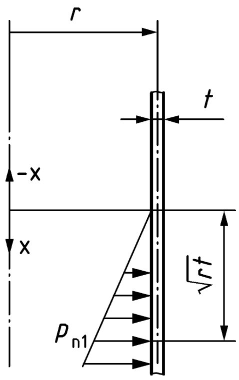


Figure C.15 — Termination of a hydrostatic internal pressure

The reference membrane theory circumferential stress is $\sigma_{MT\theta} = p_{n1}\frac{r}{t}$ where the pressure p_{n1} is the value at the distance $\sqrt{rt}$ below the point of zero pressure.

The maximum surface stress σ_s and von Mises equivalent stress σ_{eq} are given in Tables C.17 and Tables C.18.

Table C.17 — Maximum stress notation

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$k_x\ \sigma_{MT\theta}$	$k_\theta\ \sigma_{MT\theta}$	$k_\tau\sqrt{t/r}\ \sigma_{MT\theta}$	$k_{eq,s}\ \sigma_{MT\theta}$	$k_{eq,m}\ \sigma_{MT\theta}$

Table C.18 — Maximum stress values

k_x	k_θ	k_τ	$k_{eq,s}$	$k_{eq,m}$
−1,060	0,510	0,160	1,005	0,275

C.6.3 Step change of thickness

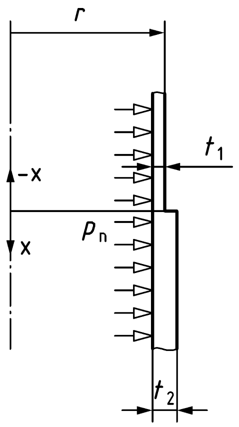


Figure C.16 — Step change of thickness under uniform internal pressure

The reference membrane theory circumferential stress is $\sigma_{MT\theta} = p_n\left(\frac{r}{t_1}\right)$.

The maximum surface stress σ_s and von Mises equivalent stress σ_{eq} are given in Tables C.19 and C.20.

Table C.19 — Maximum stress notation

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$k_x\ \sigma_{MT\theta}$	$k_\theta\ \sigma_{MT\theta}$	$k_\tau \sqrt{t/r}\ \sigma_{MT\theta}$	$k_{eq,s}\ \sigma_{MT\theta}$	$k_{eq,m}\ \sigma_{MT\theta}$

Table C.20 — Maximum stress values for different changes in thickness

$\left(\frac{t_1}{t_2}\right)$	k_x	k_θ	k_τ	$k_{eq,s}$	$k_{eq,m}$
1,0	0,0	1,0	0,0	1,0	1,0
0,8	0,0256	1,010	0,179	1,009	0,895
0,667	0,0862	1,019	0,349	1,015	0,815
0,571	0,168	1,023	0,514	1,019	0,750
0,5	0,260	1,027	0,673	1,023	0,694

Linear interpolation in $\left(\frac{t_1}{t_2}\right)$ may be used for different values of the two thicknesses.

C.7 Local ring stiffener on a cylindrical shell

C.7.1 Radial force only on the ring

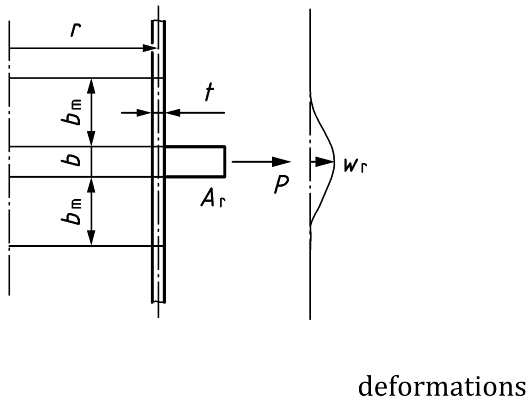


Figure C.17 — Ring attached to uniform thickness cylinder with radial load alone

(1) The stresses in the shell should be determined using the calculated value of *w* from this clause introduced into the formulae given in C.4.5.

(2) Where there is a change in the shell thickness at the ring, the method set out in 10.2.2 of EN 1993-4-1 should be used.

The reference dimension $b_{\mathrm{m}} = 0,778\sqrt{rt}$ denotes the length of participating cylinder.

The ring deflection is given by $w = w_r = \left(\frac{r}{E}\right)\left(\frac{P \cdot r}{A_r + (b + 2b_m)t}\right)$

The circumferential stress in the ring is given by $\sigma_{\theta r} = \frac{P \cdot r}{A_r + (b + 2b_m)t}$

NOTE A more detailed treatment is given in EN 1993-4-1.

C.7.2 Axial loading

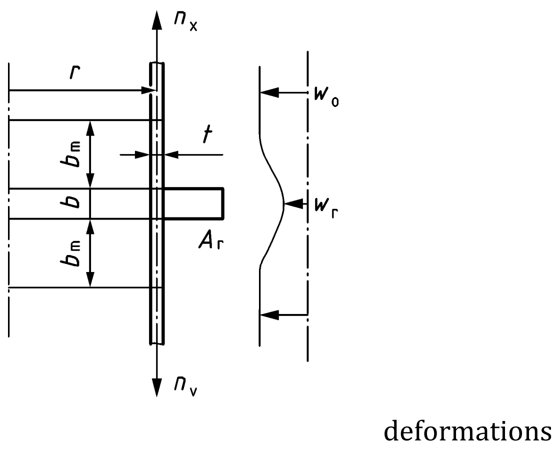


Figure C.18 — Ring attached to uniform thickness cylinder under axial load

(1) The stresses in the shell should be determined using the value of *w* calculated in this sub-clause introduced into the formulae given in C.4.5 and C.4.2.

NOTE Axial tension causes an inward displacement of the cylinder which is reduced by the ring.

The reference dimension $b_{\mathrm{m}} = 0,778\sqrt{rt}$ denotes the length of participating cylinder.

The cylinder membrane deflection is given by $w_0 = -\nu\left(\frac{r}{E}\right)\sigma_{MTx}$

The ring deflection is given by $w_r = w_0 \frac{(b+2b_m)t}{A_r + (b+2b_m)t}$

The relative deflection of the ring relative to the shell is given by

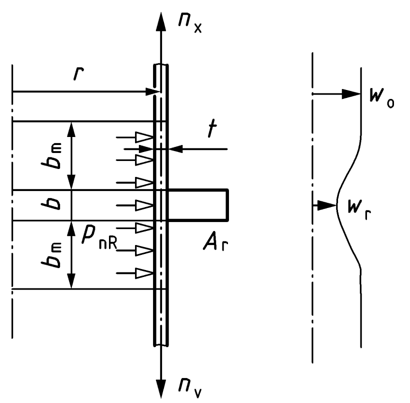
$$w = w_r - w_0 = -w_0 \frac{A_r}{A_r + (b+2b_m)t}$$

The circumferential stress in the ring is $\sigma_{\theta r} = E\frac{w_r}{r}$

The axial stress in the adjacent shell is $\sigma_{MTx} = \frac{n_x}{t}$

The bending and von Mises stresses in the shell adjacent to the ring may be found using the relative deflection *w* in the rules of C.4.5.

C.7.3 Uniform internal pressure



deformations

Figure C.19 — Ring attached to uniform thickness cylinder under uniform internal pressure

(1) The stresses in the shell should be determined the value of *w* calculated in this sub-clause introduced into the formulae given in C.4.5 and C.4.1.

Using the reference membrane theory circumferential stress $\sigma_{MT\theta} = \frac{p_n r}{t}$

The membrane radial deflection of the shell is given by $w_0 = \sigma_{MT\theta}\frac{r}{E}$

The reference dimension $b_{\mathrm{m}} = 0,778\sqrt{rt}$ denotes the length of participating cylinder.

The effect of the ring on the shell is defined by the parameter $\kappa = \frac{A_r}{A_r + (b+2b_m)t}$

The ring deflection is given by $w_r = w_0(1-\kappa)$

The deflection of the ring relative to the shell is given by $w = w_r - w_0 = -w_0\kappa$

The circumferential stress in the ring is $\sigma_{\theta r} = E\dfrac{w_r}{r}$

Table C.21 — Maximum stress notation

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$k_x\,\sigma_{MT\theta}$	$k_\theta\,\sigma_{MT\theta}$	$k_\tau\,\sqrt{t/r}\,\sigma_{MT\theta}$	$k_{eq,s}\,\sigma_{MT\theta}$	$k_{eq,m}\,\sigma_{MT\theta}$

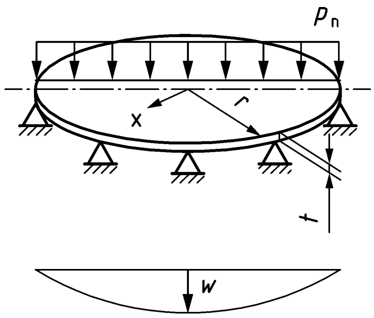
Table C.22 — Maximum stress values for different values of *κ*

κ	k_x	k_θ	k_τ	$k_{eq,s}$	$k_{eq,m}$
1,0	1,816	1,080	1,169	1,614	1,043
0,75	1,312	1,060	0,877	1,290	1,032
0,50	0,908	1,040	0,585	1,014	1,021
0,0	0,0	1,000	0,0	1,000	1,000

Linear interpolation in κ may be used for different sizes of ring.

C.8 Circular plates with simply supported boundary conditions

C.8.1 Uniform transverse load



deflected shape

Figure C.20 — Circular plate with simply supported edges under uniform transverse pressure

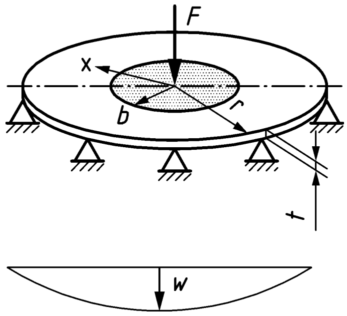
The central transverse deflection is given by $w = 0{,}696\dfrac{p_n r^4}{Et^3}$

The bending stresses are identical in the radial and circumferential directions with maximum values given by

$$\text{max. } \sigma_{bx} = \text{max. } \sigma_{b\theta} = 1{,}238\, p_n \left(\frac{r}{t}\right)^2$$

The transverse pressure at first yield in bending is given by $p_{n,y} = 0{,}808\left(\dfrac{t}{r}\right)^2 f_y$

C.8.2 Local circular distributed load



deflected shape

Figure C.21 — Circular plate with simply supported edges under local transverse pressure

Uniform pressure p_n on circular patch of radius b

The total transverse force is $F = p_n \pi b^2$ with $b < 0{,}2\ r$

The central transverse deflection is given by $w = 0{,}606 \dfrac{Fr^2}{Et^3}$

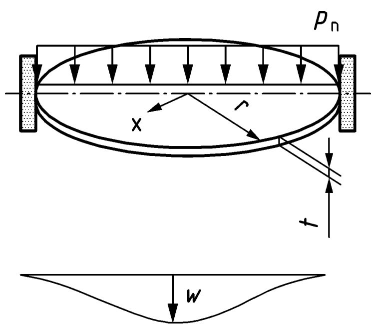
The bending stresses are identical in the radial and circumferential directions with maximum values given by

max. σ_{bx} = max. $\sigma_{b\theta}$ = $0{,}621 \dfrac{F}{t^2}\left(0{,}769 + \ln \dfrac{r}{b}\right)$

The total transverse force at first yield in bending is given by $F_y = \dfrac{1{,}611}{\left(0{,}769 + \ln \dfrac{r}{b}\right)} t^2 f_y$

C.9 Circular plates with clamped boundary conditions

C.9.1 Uniform load



deflected shape

Figure C.22 — Circular plate with clamped edges under uniform transverse pressure

The central transverse deflection is given by $w = 0{,}171 \dfrac{p_n r^4}{Et^3}$

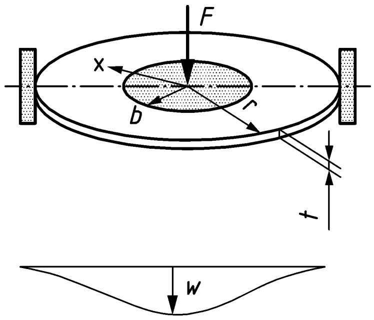
The maximum stresses are given in Table C.23 and defined in terms of $\sigma_0 = p_n (\frac{r}{t})^2$

The transverse pressure at first yield in bending at the edge is given by $p_{n,y} = 1{,}50 \left(\dfrac{t}{r}\right)^2 f_y$

Table C.23 — Maximum stress values

Maximum σ_{bx} at centre	Maximum $\sigma_{b\theta}$ at centre	Maximum σ_{eq} at centre	Maximum σ_{bx} at edge	Maximum $\sigma_{b\theta}$ at edge	Maximum σ_{eq} at edge
0,488 σ_o	0,488 σ_o	0,488 σ_o	0,75 σ_o	0,225 σ_o	0,667 σ_o

C.9.2 Plate with fixed boundary: local distributed load



deflected shape

Figure C.23 — Circular plate with clamped edges under local transverse pressure

Uniform pressure p_n on circular patch of radius b

The total transverse force is $F = p_n \pi b^2$ with $b < 0{,}2\,r$

The central transverse deflection is given by $w = 0{,}217\left(\frac{Fr^2}{Et^3}\right)$

The bending stresses are identical in the radial and circumferential directions with maximum values

max. σ_{bx} = max. $\sigma_{b\theta}$ = $0{,}621\left(\frac{F}{t^2}\right)\left(0{,}769 + \ln\frac{r}{b}\right)$

The total transverse force at first yield in bending at the centre is given by $F_y = 1{,}611\frac{t^2}{(0{,}769 + \ln\frac{r}{b})}f_y$

The maximum stresses at the centre are given in Table C.24 and defined in terms of $\sigma_0 = \frac{F}{t^2}$

Table C.24 — Maximum bending and von Mises stress values

Maximum σ_{bx} at centre	Maximum $\sigma_{b\theta}$ at centre	Maximum σ_{eq} at centre	Maximum σ_{bx} at edge	Maximum $\sigma_{b\theta}$ at edge	Maximum σ_{eq} at edge
$0{,}621\left(\ln\frac{r}{b}\right)\sigma_0$	$0{,}621\left(\ln\frac{r}{b}\right)\sigma_0$	$0{,}621\left(\ln\frac{r}{b}\right)\sigma_0$	0,477 σ_o	0,143 σ_o	0,424 σ_o

Annex D
(normative)

Formulae to determine the buckling resistance of unstiffened shells when using stress design

D.1 Use of this annex

(1) This Normative Annex contains additional formulae to determine the buckling resistance of unstiffened shells when using stress design.

D.2 Scope and field of application

(1) This Normative Annex gives formulae to determine the buckling resistance of unstiffened shells when using stress design.

D.3 Cylindrical shells of constant wall thickness: basic load cases

D.3.1 Notation and boundary conditions

(1) Geometrical quantities:

L cylinder length or segment length between defined boundaries;

r radius of cylinder middle surface;

t thickness of shell;

δ_0 imperfection amplitude used in a design calculation.

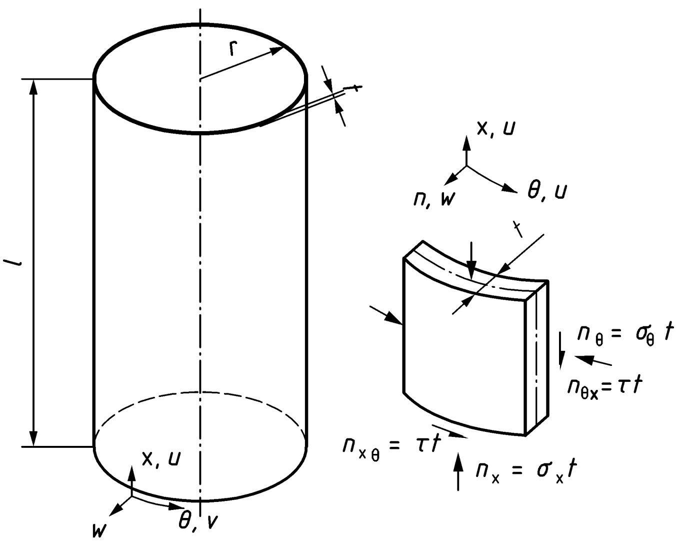


Figure D.1 — Cylinder geometry, membrane stresses and stress resultants

(2) The relevant boundary conditions are set out in 4.3, 6.2.2 and 9.3.

D.3.2 Dimensionless lengths

(1) The length of the shell segment L is characterised in terms of two relative length parameters. The first relative length ω is given by:

$$\omega = \frac{L}{r}\sqrt{\frac{r}{t}} = \frac{L}{\sqrt{rt}} \tag{D.1}$$

The second relative length Ω is given by:

$$\Omega = \frac{L}{r}\sqrt{\frac{t}{r}} = \frac{t}{r}\omega \tag{D.2}$$

D.3.3 Axial (meridional) compression

D.3.3.1 Length domains

(1) Under axial compression, cylinders are classed as short if:

$$\omega < 1{,}7 \tag{D.3}$$

(2) Under axial compression, cylinders are classed as of medium-length if:

$$1{,}7 \leq \omega \leq 1{,}43\ \frac{r}{t} \tag{D.4}$$

(3) Under axial compression, cylinders are classed as long if:

$$\omega > 1{,}43\ \frac{r}{t} \tag{D.5}$$

NOTE: The value 1,43 is based on the assumption that the effective length of the shell or tube in buckling as a structural member involves an effective length of 2, since most practical long shell structures of this kind are in the form of a cantilever.

D.3.3.2 Critical axial buckling stresses

(1) The following formulae may only be used for shells with boundary conditions BC 1 or BC 2 at both edges.

(2) The elastic critical axial buckling stress should be obtained from:

$$\sigma_{x,Rcr} = 0{,}605 E C_x \frac{t}{r} \tag{D.6}$$

(3) For medium-length cylinders, the factor C_x should be taken as:

$$C_x = 1{,}0 \tag{D.7}$$

(4) For short cylinders the factor C_x should be taken as:

$$C_x = 1{,}36 - \frac{1{,}83}{\omega} + \frac{2{,}07}{\omega^2} \tag{D.8}$$

NOTE Long cylinders are now defined as those subject to Euler buckling, since recent evidence shows that the resistance of an imperfect cylinder is unaffected by the drop in critical buckling stress as lower modes occur in classical LBA calculations. The definition of “long” is now ω > 1.43 (r/t).

(5) Long cylinders should be checked for local buckling using the rules for medium length cylinders.

(6) Long cylinders should also be checked for column buckling using the rules of EN 1993-1-1.

(7) Cylinders need not be checked against axial compression shell buckling if they satisfy:

$$\frac{r}{t} \leq \frac{1}{165} C_x \frac{E}{f_{yk}} \tag{D.9}$$

NOTE The value of the coefficient 165 is directly related to the value of $\bar{\lambda}_{x0}$ = 0,10 in Formula (D.10).

(8) Cylinders under a combination of uniform axial compression and uniform bending may be treated using the provisions of Annex E, E.3.3.

D.3.3.3 Axial compression buckling capacity parameters

(1) The axial squash limit relative slenderness $\bar{\lambda}_{x0}$ should be taken as:

$$\bar{\lambda}_{x0} = 0,10 \tag{D.10}$$

(2) The axial elastic imperfection reduction factor α_x should be obtained from:

$$\alpha_x = \alpha_{xG} \alpha_{xI} \tag{D.11}$$

$$\alpha_{xG} = 0,83 \tag{D.12}$$

$$\alpha_{xI} = \frac{1}{1 + 2,2\left(\delta_0 / t\right)^{0,75}} \tag{D.13}$$

in which δ_0 is the imperfection amplitude given by:

$$\frac{\delta_0}{t} = \frac{1}{Q_x}\sqrt{\frac{r}{t}} \tag{D.14}$$

where

Q_x is the axial compression fabrication quality parameter.

(3) The fabrication quality parameter Q_x should be taken from Table D.1 for the specified fabrication tolerance quality class.

Table D.1 — Values of axial compression fabrication quality parameter Q_x

Fabrication tolerance quality class	Description	Q_x
Class A	Excellent	40
Class B	High	25
Class C	Normal	16

(4) The plastic range factor β_x should be taken as:

$$\beta_x = 1 - \frac{0,75}{1 + 1,1(\delta_0 / t)} \tag{D.15}$$

(5) The interaction exponent η_x should be obtained from Formula (D.18) with the two values η_{x0} and η_{xp} taken as:

$$\eta_{x0} = 1{,}35 - 0{,}10\left(\delta_0/t\right) \tag{D.16}$$

$$\eta_{xp} = \frac{1}{0{,}45 + 0.72\left(\delta_0/t\right)} \tag{D.17}$$

with

$$\eta_x = \left[\frac{\bar{\lambda}_x\left(\eta_{xp} - \eta_{xo}\right) + \bar{\lambda}_{xp}\eta_{xo} - \bar{\lambda}_{xo}\eta_{xp}}{\bar{\lambda}_{xp} - \bar{\lambda}_{xo}}\right] \tag{D.18}$$

(6) The hardening limit χ_{xh} should be taken as:

$$\chi_{xh} = 1{,}10 \tag{D.19}$$

D.3.3.4 Stainless steel cylinders under axial compression

(1) With the exceptions defined here, the formulae of D.3.3.3 may be applied to shells constructed from austenitic, austenitic-ferritic (duplex) and ferritic stainless steels under axial compression and with all the boundary conditions of D.3.1 (2).

(2) In the elastic range, the buckling resistance of a cylindrical shell made of stainless steel is comparable with those made of carbon steel. The axial elastic imperfection reduction factor $\alpha_x = \alpha_{xG}\,\alpha_{xI}$ should therefore be obtained from Formulae (D.11) to (D.13).

NOTE The rounded character of the stress-strain response for stainless steels has no influence on elastic buckling and the imperfection amplitudes are expected to be similar.

(3) The following buckling design rules are applicable to cylindrical shells made from stainless steels that meet the following ratios of the parameter $E/R_{p,0.2}$:

— austenitic stainless steel shells with $E/R_{p,0.2} \le 870$;

— austenitic-ferritic stainless steel shells with $E/R_{p,0.2} \le 400$;

— ferritic stainless steel shells with $E/R_{p,0.2} \le 715$.

where

E is the initial tangent value of Young's modulus;

$R_{p,0.2}$ is the 0,2% proof stress (see 5.1).

NOTE Most steels covered by EN 1993-1-4 are included within these restrictions.

(4) All other capacity parameters for austenitic, austenitic-ferritic and ferritic stainless steel shells at ambient temperatures should be taken from Tables D.2 and D.3 for the three Fabrication Tolerance Quality Classes A to C. The fabrication quality parameter Q should be obtained from Table D.1 for each fabrication tolerance quality class and the imperfection amplitude found using Formula (D.14).

Table D.2 — Values of the capacity parameters χ_{xh}, $\bar{\lambda}_{x0}$, μ_{x0} and μ_{xh} for cylindrical shells made of austenitic, austenitic-ferritic and ferritic steels

Stainless steel type	**Hardening limit**	**Squash limit relative slenderness**	**Plastic interaction exponents**	
	χ_{xh}	$\bar{\lambda}_{x0}$	μ_{x0}	μ_{xh}
Austenitic	1,2	0,31	1,50	1,10
Duplex		0,36	1,08	0,60
Ferritic		0,30	1,125	0,70

Table D.3 — Formulae for the capacity parameters β_x, η_{x0} and η_{xp} for cylindrical shells made of austenitic, austenitic-ferritic and ferritic steels

Stainless steel type	**Plastic range factor**	**Elastic-plastic interaction exponents**	
	β_x	η_{x0}	η_{xp}
Austenitic	$1-\dfrac{0,44}{1+0,955(\delta_0/t)^{0,96}}$	$0,85-0,51(\delta_0/t)^{0,71}$	$\dfrac{0,68}{1+0,56(\delta_0/t)}$
Duplex	$1-\dfrac{0,48}{1+1,35(\delta_0/t)^{0,95}}$	$1,1-0,631(\delta_0/t)^{0,62}$	$\dfrac{0,97}{1+1,06(\delta_0/t)}$
Ferritic	$1-\dfrac{0,66}{1+1,13(\delta_0/t)^{0,87}}$	$1,05-0,601(\delta_0/t)^{0,62}$	$\dfrac{1,04}{1+1,11(\delta_0/t)}$

(5) The elastic-plastic buckling reduction factor χ_x for stocky stainless steel shells at slendernesses below $\bar{\lambda}_{x0}$ should be determined as a function of the relative slenderness $\bar{\lambda}_x$ of the shell using:

$$\chi_x = \chi_{xh} - (\chi_{xh} - 1)(\bar{\lambda}_x / \bar{\lambda}_{x0})^{\mu_x} \qquad \text{when } \bar{\lambda}_x \le \bar{\lambda}_{x0} \tag{D.20}$$

(6) The value of plastic interaction exponent μ_x is defined by two limiting values μ_{x0} and μ_{xh}, with η_{xh} determined as:

$$\mu_x = \left[\mu_{xh} + \left(\frac{\bar{\lambda}_x}{\bar{\lambda}_{x0}}\right)(\mu_{x0} - \mu_{xh})\right] \quad \text{but } \mu_x \ge 1 \tag{D.21}$$

in which μ_{x0} is the value of μ_x at $\bar{\lambda}_x = \bar{\lambda}_{x0}$ and μ_{xh} is the value of μ_x at $\bar{\lambda}_x = 0$.

NOTE When the buckling resistance of a shell with a nonlinear stress-strain curve is assessed using formulae given for carbon steel (Formulae (D.10) to (D.19)) with a modified modulus taken as the secant value at the 0,2% proof stress, the outcome can produce very unconservative buckling resistances at low slendernesses and very conservative resistances at higher slendernesses (see 5.1 (6)-(8)).

D.3.4 Circumferential (hoop) compression

D.3.4.1 Length domains

(1) Under circumferential compression, medium-length cylinders are defined by:

$$\omega_s \leq \omega \leq 1{,}63\ C_\theta \frac{r}{t} \tag{D.22}$$

in which C_θ is given in Table D.4 and ω_S is given in Table D.5.

(2) Under circumferential compression, a cylinder is classed as short if its dimensionless length ω lies below ω_S, the value of which depends on the boundary conditions as given in Table D.5.

(3) Under circumferential compression, cylinders are classed as long if:

$$\omega > 1{,}63\ C_\theta \frac{r}{t} \tag{D.23}$$

Table D.4 — External pressure buckling factors for medium-length cylinders C_θ

Cylinder end	Boundary condition	Value of C_θ
end 1 end 2	BC 1 BC 1	1,5
end 1 end 2	BC 1 BC 2	1,25
end 1 end 2	BC 2 BC 2	1,0
end 1 end 2	BC 1 BC 3	0,6
end 1 end 2	BC2 BC3	See Table D.5
end 1 end 2	BC 3 BC 3	0

Table D.5 — External pressure buckling factors for short cylinders $C_{\theta s}$

Boundary condition at each end		**Formula for $C_{\theta s}$**	**Value of ω_s** at the lower limit of validity of Table D.4
End 1	**End 2**		
BC1r	BC1r	$1,50-\frac{1,64}{\omega}+\frac{8,7}{\omega^2}+\frac{24,2}{\omega^3}$	110 *
BC1r	BC1f	$1,50-\frac{1,9}{\omega}+\frac{8,9}{\omega^2}+\frac{0,9}{\omega^3}$	110 *
BC1f	BC1f	$1,50-\frac{2}{\omega}+\frac{5,1}{\omega^2}+\frac{2,76}{\omega^3}$	125 *
BC1r	BC2r	$1,25+\frac{0.86}{\omega}+\frac{2,6}{\omega^2}+\frac{27,8}{\omega^3}$	65
BC1r	BC2f	$1,25+\frac{5,8}{\omega^2}+\frac{2,8}{\omega^3}$	25
BC1f	BC2r	$1,25+\frac{0.82}{\omega}-\frac{0,84}{\omega^2}+\frac{18,3}{\omega^3}$	45
BC1f	BC2f	$1,25+\frac{1,9}{\omega^2}+\frac{2,9}{\omega^3}$	12
BC2r	BC2r	$1+\frac{2,6}{\omega}-\frac{1,6}{\omega^2}+\frac{30,4}{\omega^3}$	125
BC2r	BC2f	$1+\frac{1,8}{\omega}+\frac{0,1}{\omega^2}+\frac{9,3}{\omega^3}$	125
BC2f	BC2f	$1+\frac{1,3}{\omega}-\frac{0,8}{\omega^2}+\frac{6,9}{\omega^3}$	125
BC1r	BC3f	$0,6+\frac{0,77}{\omega^2}$	11
BC1f	BC3f	0,60	Both short and medium lengths
BC2r	BC3f	$0,05+\frac{1,8}{\omega}-\frac{2,6}{\omega^2}+\frac{2,6}{\omega^3}$	Both short and medium lengths
BC2f	BC3f	$\frac{0,34}{\omega}+\frac{0,27}{\omega^2}-\frac{0,25}{\omega^3}$ $-0,3\sqrt{\frac{t}{r}}\left(0,33-\omega\sqrt{\frac{t}{r}}\right)$	Both short and medium lengths
where $\omega=\frac{L}{\sqrt{rt}}$ Use of Table D.4 is unsafe for values of ω below the value ω_s marked *			

D.3.4.2 Critical circumferential buckling stresses

(1) The following formulae may be applied to shells with all the boundary condition combinations identified here.

(2) For medium length cylinders the elastic critical circumferential buckling stress should be obtained from:

$$\sigma_{\theta,Rcr} = 0,92E\left(\frac{C_\theta}{\omega}\right)\left(\frac{t}{r}\right) \tag{D.24}$$

The factor C_θ should be taken from Table D.4 with a value that depends on the boundary conditions, see 6.2.2.2 and 9.3.

(3) For short cylinders, the elastic critical circumferential buckling stress should be obtained from:

$$\sigma_{\theta,Rcr} = 0,92E\left(\frac{C_{\theta s}}{\omega}\right)\left(\frac{t}{r}\right) \tag{D.25}$$

The factor $C_{\theta s}$ should be taken from Table D.5 with a value that depends on the dimensionless length ω and the boundary conditions, see 6.2.2.2 and 9.3.

NOTE For all boundary conditions in short cylinders except those with boundary conditions BC1 at both ends, Table D.4 also gives a safe estimate of the value of $C_{\theta s}$. Table D.4 gives unsafe values of $C_{\theta s}$ for three short cylinder arrangements with boundary conditions BC1 at both ends.

(4) For long cylinders the elastic critical circumferential buckling stress should be obtained from:

$$\sigma_{\theta,Rcr} = E\left(\frac{t}{r}\right)^2\left[0,275 + 2,03\left(\frac{C_\theta}{\omega}\cdot\frac{r}{t}\right)^4\right] \tag{D.26}$$

where C_θ is as defined in Table D.4.

D.3.4.3 Circumferential buckling capacity parameters

(1) The circumferential elastic imperfection reduction factor α_θ should be obtained from:

$$\alpha_\theta = \alpha_{\theta G}\alpha_{\theta I} \tag{D.27}$$

$$\alpha_{\theta G} = 0,95 \tag{D.28}$$

$$\alpha_{\theta I} = \frac{1}{1 + b\left(\delta_0/t\right)^{0,8}} \tag{D.29}$$

with

$$b = 9,8\left(\frac{r}{L}\right)^{0,75}\left(\frac{t}{r}\right)^{0,40} \tag{D.30}$$

in which δ_0 is the imperfection amplitude given by:

$$\frac{\delta_{0\theta}}{t} = \frac{1}{Q_\theta}\left(\frac{L}{r}\right)^{0.8}\left(\frac{r}{t}\right)^{0.5} \tag{D.31}$$

where

Q_θ is the circumferential compression fabrication quality parameter.

NOTE This description of the assumed imperfection is aligned with the use of the gauge length $\ell_{g\theta}$ used in the circumferential tolerance measurement in 9.4.

(3) The fabrication quality parameter Q_θ should be taken from Table D.6 for the specified fabrication tolerance quality class.

Table D.6 — Values of circumferential compression fabrication quality parameter Q_θ

Fabrication tolerance quality class	**Description**	Q_θ
Class A	Excellent	75
Class B	High	40
Class C	Normal	20

(2) The circumferential squash limit relative slenderness $\overline{\lambda}_{\theta 0}$ should be taken as:

$$\overline{\lambda}_{\theta 0} = 0,40 \quad \text{(D.32)}$$

(3) The circumferential plastic range factor β_θ, should be taken as:

$$\beta_\theta = 0,60 \quad \text{(D.33)}$$

(4) The circumferential interaction exponent η_θ should be taken as:

$$\eta_\theta = 1,00 \quad \text{(D.34)}$$

(5) The circumferential hardening limit $\chi_{\theta h}$ should be taken as:

$$\chi_{\theta h} = 1,10 \quad \text{(D.35)}$$

(6) Cylinders need not be checked against circumferential shell buckling if they satisfy:

$$\frac{r}{t} \leq 0,21 \sqrt{\frac{E}{f_{yk}}} \quad \text{(D.36)}$$

D.3.5 Shear (torsion)

D.3.5.1 Length domains

(1) Under pure membrane shear cylinders are defined as short if:

$$\omega < 10 \quad \text{(D.37)}$$

(2) Under pure membrane shear cylinders are defined as of medium-length if:

$$10 \leq \omega \leq 8,7 \frac{r}{t} \quad \text{(D.38)}$$

(3) Under pure membrane shear cylinders are defined as long if:

$$\omega > 8,7 \frac{r}{t} \quad \text{(D.39)}$$

NOTE The condition of torsion provides the only load case where a cylinder is subjected to pure shear. Whilst torsional loading can occur in practical shells, this case is uncommon, and is here used to provide the third condition of a pure membrane stress acting on the cylinder. It is expected to be used in combination with other stress resultants, and in these cases this treatment is often very conservative.

D.3.5.2 Critical shear buckling stresses

(1) The following formulae should be applied only to shells with boundary conditions BC1 or BC2 at both edges.

(2) The elastic critical shear buckling stress should be obtained from:

$$\tau_{x\theta,Rcr} = 0,75 E C_\tau \sqrt{\frac{1}{\omega}}\left(\frac{t}{r}\right) \tag{D.40}$$

(3) The factor C_τ for medium-length cylinders should be taken as:

$$C_\tau = 1,0 \tag{D.41}$$

(4) The factor $C_{\tau s}$ for short cylinders should be obtained from:

$$C_{\tau s} = \sqrt{1 + \frac{a_{\tau s}}{\omega^b}} \tag{D.42}$$

with $a_{\tau s} = 120 - \dfrac{130}{1 + 0,015\dfrac{r}{t}}$ for BC1r or BC2r boundary conditions (D.43)

or $a_{\tau s} = 70 - \dfrac{75}{1 + 0,015\left(\dfrac{r}{t}\right)^{1,1}}$ for BC1f or BC2f boundary conditions (D.44)

and

$$b = 3 - \frac{5}{1 + 0,4\left(\dfrac{r}{t}\right)^{0.6}} \tag{D.45}$$

(5) The factor C_τ for long cylinders should be determined from:

$$C_{\tau L} = \frac{1}{3}\sqrt{\omega \frac{t}{r}} \tag{D.46}$$

D.3.5.3 Shear buckling capacity parameters

(1) The shear geometric reduction factor $\alpha_{\tau G}$ should be taken as

$$\alpha_{\tau G} = 0,96 \tag{D.47}$$

(2) The shear elastic imperfection reduction factor $\alpha_\tau = \alpha_{\tau G}\alpha_{\tau I}$ should be taken as:

$$\alpha_{\tau I} = \frac{1}{1 + 0,5(\delta_0/t)} \tag{D.48}$$

in which δ_k is the shear imperfection amplitude given by:

$$\frac{\delta_0}{t} = \frac{1}{Q_\tau}\sqrt{\frac{r}{t}} \tag{D.49}$$

where

Q_τ is the shear fabrication quality parameter.

(3) The fabrication quality parameter Q_τ should be taken from Table D.7 for the specified fabrication tolerance quality class.

Table D.7 — Values of shear fabrication quality parameter Q_τ

Fabrication tolerance quality class	**Description**	Q_τ
Class A	Excellent	40
Class B	High	25
Class C	Normal	16

(4) The shear squash limit slenderness $\bar{\lambda}_{\tau 0}$ should be taken as:

$$\bar{\lambda}_{\tau 0} = 0,40 \tag{D.50}$$

(5) The shear plastic range factor β_τ, should be taken as:

$$\beta_\tau = 0,60 \tag{D.51}$$

(6) The shear interaction exponent η_τ should be taken as:

$$\eta_\tau = 1,0 \tag{D.52}$$

(7) The shear hardening limit $\chi_{\tau h}$ should be taken as:

$$\chi_{\tau h} = 1,0 \tag{D.53}$$

(8) Cylinders need not be checked against shear buckling if they satisfy:

$$\frac{r}{t} \leq 0,17\left[\frac{E}{f_{yk}}\right]^{0.67} \tag{D.54}$$

D.4 Cylindrical shells of constant wall thickness: combined cases

D.4.1 Axial (meridional) compression with coexistent internal pressure

D.4.1.1 Pressurised critical axial buckling stress

(1) The elastic critical axial buckling stress $\sigma_{x,Rcr}$ may be assumed to be unaffected by the presence of internal pressure and may be obtained as specified in D.3.3.2.

D.4.1.2 Pressurised axial buckling capacity parameters

(1) The pressurised axial buckling stress should be verified analogously to the unpressurised axial buckling stress as specified in 9.5 and D.3.3.2. However, the unpressurised elastic buckling reduction factor α_x should be replaced by the pressurized elastic buckling reduction factor α_{xp}.

(2) The pressurized elastic buckling reduction factor α_{xp} should be taken as the smaller of the two following values:

α_{xpe} is a factor covering pressure-induced elastic stabilization;

α_{xpp} is a factor covering pressure-induced plastic destabilization.

(3) The imperfection reduction factor α_{xpe} should be obtained from:

$$\alpha_{xpe} = \alpha_x + (1-\alpha_x)\left[\frac{\overline{p}_s}{\overline{p}_s + 0{,}3/\alpha_x^{0,5}}\right] \tag{D.55}$$

$$\overline{p}_s = \left(\frac{p_s}{\sigma_{x,Rcr}}\right)\left(\frac{r}{t}\right) \tag{D.56}$$

where

p_s is the smallest design value of local internal pressure at the location of the point being assessed, guaranteed to coexist with the axial compression;

α_x is the unpressurised axial elastic buckling reduction factor according to D.3.3.3 (2);

$\sigma_{x,Rcr}$ is the elastic critical axial buckling stress according to D.3.3.2 (2).

(4) The factor α_{xpe} should not be applied to cylinders that are long according to D.3.3.1 (3). In addition, it should not be applied unless one of the following two conditions are met:

— the cylinder is medium-length according to D.3.3.1 (2);

— the cylinder is short according to D.3.3.1 (1) and $C_x = 1$ has been adopted in D.3.3.2 (2).

(5) The imperfection reduction factor α_{xpp} should be taken as:

$$\alpha_{xpp} = \left\{1 - \left(\frac{\overline{p}_g}{\overline{\lambda}_x^2}\right)^2\right\}\left[1 - \frac{1}{1{,}12 + s^{3/2}}\right]\left[\frac{s^2 + 1{,}21\overline{\lambda}_x^2}{s(s+1)}\right] \tag{D.57}$$

$$\overline{p}_g = \left(\frac{p_g}{\sigma_{x,Rcr}}\right)\left(\frac{r}{t}\right) \tag{D.58}$$

$$s = \frac{1}{400}\cdot\frac{r}{t} \tag{D.59}$$

where

p_g is the largest design value of local internal pressure at the location of the point being assessed that can coexist with the axial compression;

$\overline{\lambda}_x$ is the dimensionless shell slenderness parameter according to 9.5.2 (3);

$\sigma_{x,Rcr}$ is the elastic critical axial buckling stress according to D.3.3.2 (2).

D.4.2 External pressure under a wind pressure distribution

D.4.2.1 Critical circumferential buckling pressure under wind

(1) Where the uniform thickness shell is subject to a wind pressure distribution as shown in Figure D.2, the critical buckling resistance should be determined as follows.

(2) The upper boundary of the shell is assumed to be held circular, either by attachment to a roof or by a ring stiffener of appropriate size.

(3) The reference critical uniform pressure should be calculated as:

$$q_{Rcr} = 0{,}92 \frac{E}{\omega}\left(\frac{t}{r}\right)^{2} \tag{D.60}$$

(4) For the wind assessment, the relative length of the shell is defined as:

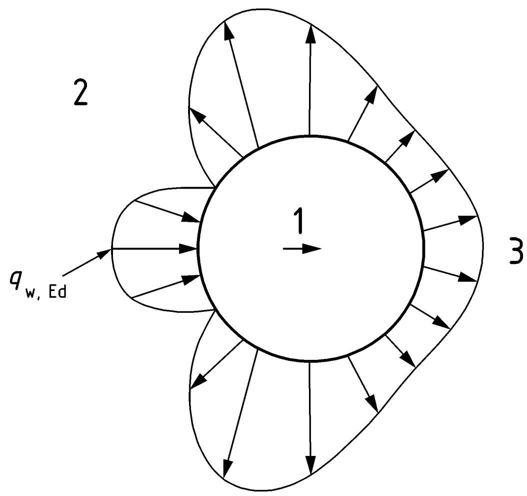
$$\Omega = \left(\frac{L}{r}\right)\left(\frac{t}{r}\right)^{1/2} \tag{D.61}$$

(5) The critical wind stagnation pressure is given by:

$$q_{w,Rcr} = q_{Rcr}\left\{0{,}83 + 1{,}64\Omega^{0{,}23}\right\} \qquad \text{when } \Omega < 0{,}40 \tag{D.62}$$

$$q_{w,Rcr} = q_{Rcr}\left\{0{,}55 + 0{,}705\Omega^{-0.9}\right\} \qquad \text{when } 0{,}40 \le \Omega < 1{,}40 \tag{D.63}$$

$$q_{w,Rcr} = 1{,}07 q_{Rcr} \qquad \text{when } 1{,}40 \le \Omega \tag{D.64}$$



Key

1 wind direction

2 windward side

3 leeward side

Figure D.2 — Wind pressure distribution

(6) The relative length that defines the effect of geometric nonlinearity should be calculated as:

$$\xi = \left(\frac{L}{r}\right)\left(\frac{t}{r}\right)^{4/7} \tag{D.65}$$

(7) The sensitivity to geometric nonlinearity α_G may be taken as:

$$\alpha_{\theta G,w} = 1,0 \qquad \text{when } \xi \leq 0,161 \tag{D.66}$$

$$\alpha_{\theta G,w} = \left(\frac{0,1}{2,12\xi^{0,06} - 1,8} \right) \qquad \text{when } 0,161 < \xi < 0,344 \tag{D.67}$$

$$\alpha_{\theta G,w} = 0,53 \qquad \text{when } 0,334 \leq \xi \tag{D.68}$$

(8) The imperfection reduction factor $\alpha_{\theta I}$ should be found using Formula (D.29), together with the appropriate Fabrication Quality Class as defined in Table D.6.

(9) The characteristic value of the buckling pressure should be found as:

$$q_{w,Rk} = \alpha_{\theta I} \alpha_{\theta G,w} \, q_{w,Rcr} \tag{D.69}$$

(10) The design value of the stagnation pressure of the wind at the eaves should be chosen as $q_{w,Ed}$

(11) The total inward pressure at the stagnation location should be evaluated as:

$$q_{net,Ed} = q_{w,Ed} + q_{s,Ed} \tag{D.70}$$

where

$q_{w,Ed}$	is the stagnation pressure at the windward location (Figure D.2);
$q_{s,Ed}$	is the internal suction caused by venting, internal partial vacuum or other phenomena.

(12) The following check should be made

$$q_{net,Ed} \leq q_{w,Rk} / \gamma_{M1} \tag{D.71}$$

(13) Formula (D.25) and Table D.4 should not be used in conjunction with D.4.2.1.

NOTE This process in D.4.2.1 is valid for short and medium length cylindrical shells and conservative for long cylinders. If the higher resistance for short cylinders under uniform external pressure (Formula (D.25)) is included in the buckling assessment, the same short cylinder effects are included twice, making the result unsafe.

(14) Where this load case is required to be combined with other stress components, the circumferential design stress to be introduced into 9.5 should be determined as:

$$\sigma_{\theta,Ed} = q_{net,Ed} \left(\frac{r}{t} \right) \tag{D.72}$$

D.4.3 Combinations of axial (meridional) compression, circumferential (hoop) compression and shear

(1) The buckling interaction parameters to be used in 9.5.3 (3) may be obtained from:

$$k_{ix} = 1,25 + 0,75\, \chi_x \tag{D.73}$$

$$k_{i\theta} = 1,25 + 0,75\, \chi_\theta \tag{D.74}$$

$$k_{i\tau} = 1,75 + 0,25\, \chi_\tau \tag{D.75}$$

$$a_i = (\chi_x \chi_\theta)^2 \tag{D.76}$$

where

$\chi_x, \chi_\theta, \chi_\tau$ are the elastic-plastic buckling reduction factors defined in 9.5.2, using the buckling parameters given in D.3.2 to D.3.5.

NOTE 1 This treatment of the interaction between the three membrane stress resultants in different directions is based on analyses of cylindrical shells under the combination with uniform stresses throughout the shell. It is therefore likely to be very conservative where it is applied to a local membrane stress state at a single location in a shell (the commonest situation where multiple stresses are all present).

NOTE 2 The buckling resistance depends very much on the size of the buckling mode, but modes dominated by axial compression are very local, those dominated by external pressure are very large, and those dominated by shear lie in between.

(2) The three membrane stress components should be deemed to interact in combination at any point in the shell, except those adjacent to the boundaries. The buckling interaction check may be omitted for all points that lie within the boundary zone length ℓ_R adjacent to either end of the cylindrical segment. The value of ℓ_R is the smaller of:

$$\ell_R = 0{,}1L \tag{D.77}$$

and

$$\ell_R \leq 0{,}16r\sqrt{rt} \tag{D.78}$$

(3) Where checks of the buckling interaction at all points is found to be onerous, the following provisions of (4) and (5) permit a simpler conservative assessment. If the maximum value of any of the buckling-relevant membrane stresses in a cylindrical shell occurs in a boundary zone of length ℓ_R adjacent to either end of the cylinder, the interaction check of 9.5.3 (3) may be undertaken using the values defined in (4).

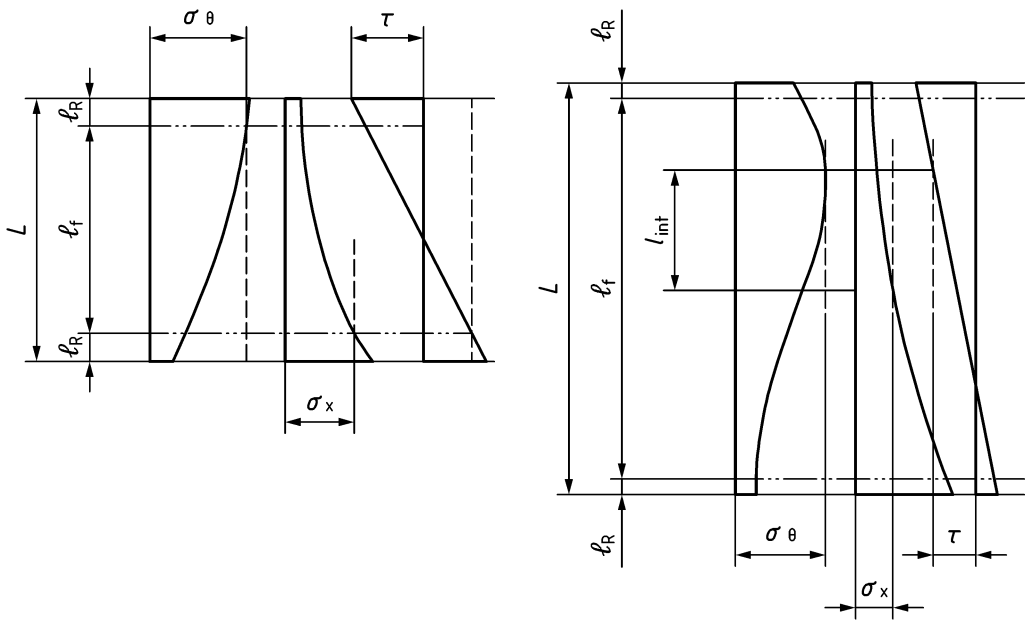
(4) Where the conditions of (3) are met, the maximum value of each of the buckling-relevant membrane stresses occurring within the free length ℓ_f (that is, outside the boundary zones, see Figure D.3a) may be used in the interaction check of 9.5.3 (3), with the free length defined as:

$$\ell_f = L - 2\ell_R \tag{D.79}$$

NOTE This treatment of the interaction between the peak values of the three membrane stress resultants in different directions and found at different locations in the shell can be very conservative indeed. In particular, it is unwise to use it where global bending is induced by shear (e.g. a tank under seismic loading), as the combination of shear in one location (treated as uniform torsion) and axial compression at a very different location, leads to a very conservative outcome.

(5) For long cylinders as defined in D.3.3.1 (3), the interaction-relevant groups introduced into the interaction check may be restricted further than the provisions of paragraphs (3) and (4). The stresses deemed to be in interaction-relevant groups may then be restricted to any region of length ℓ_{int} falling within the free remaining length ℓ_f for the interaction check, see Figure D.3 b), with the region length defined as:

$$\ell_{\text{int}} = 1{,}3r\sqrt{rt} \tag{D.80}$$



a) in a short cylinder b) in a long cylinder

Figure D.3 — Examples of interaction-relevant groups of membrane stress components

(6) If (3)-(5) above do not provide specific provisions for defining the relative locations or separations of interaction-relevant groups of membrane stress components, and a conservative treatment is still required, the maximum value of each membrane stress, irrespective of location in the shell, may be adopted into Formulae (9.33), (9.34) and (9.35), as appropriate.

NOTE This procedure has not be proven to be conservative, but it has been in use for a long time, so it has the evidential proof of success in practice.

D.5 Cylindrical shells of stepwise variable wall thickness

D.5.1 General

D.5.1.1 Notation and boundary conditions

(1) In this Clause the following notation is used:

L overall cylinder length;

r radius of cylinder middle surface;

i an integer index denoting the individual cylinder segments with constant wall thickness (from $j = 1$ to $j = n$);

t_j the constant wall thickness of segment j of the cylinder;

ℓ_j the length of segment j of the cylinder.

(2) The following formulae should only be used for shells where the two edges each has a boundary condition of either BC 1 or BC 2 (see 6.2.2.2 and 9.3).

D.5.1.2 Geometry and joint offsets

(1) Provided that the wall thickness of the cylinder increases progressively stepwise from top to bottom (see Figure D.5), the procedures given in this subclause D.5 may be used.

(2) For cylinders with overlapping joints (lap joints), the provisions for lap-jointed construction given in D.6 should be used.

(3) Intended offsets e_0 between plates of adjacent butt-jointed segments (see Figure D.4) may be treated as covered by the following formulae provided that the intended value e_0 is less than the permissible value $e_{0,p}$ which should be taken as the smaller of:

$$e_{0,p} = 0,5\ (t_{max} - t_{min}) \tag{D.81}$$

and

$$e_{0,p} = 0,5\ t_{min} \tag{D.82}$$

where

t_{max} is the thickness of the thicker plate at the joint;

t_{min} is the thickness of the thinner plate at the joint.

NOTE These restrictions correspond to a) a smooth surface on one side (D.81), and b) a limitation that the thicker plate is not thicker than twice the thickness of the thinner plate (D.82). These limitations do not generally limit practical designs.

(4) For cylinders with permissible intended offsets between plates of adjacent segments according to (3), the radius r may be taken as the mean value of all segments.

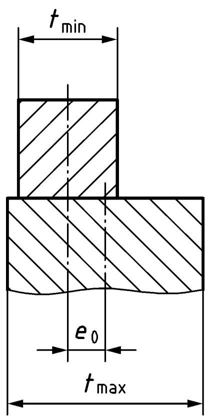


Figure D.4 — Intended offset e_0 in a butt-jointed shell

D.5.2 Axial (meridional) compression

(1) Each cylinder segment j of length ℓ_j should be treated as an equivalent cylinder of overall length $L = \ell_j$ and of uniform wall thickness $t = t_j$ according to D.3.3.

(2) For long equivalent cylinders, as governed by D.3.3.1 (3), see D.3.3.2 (5) and (6).

D.5.3 Circumferential (hoop) compression

D.5.3.1 Critical circumferential buckling pressure and stresses

(1) The wall thickness of each section of a stepped wall cylinder is assumed to increase progressively from the top to the base.

NOTE This procedure assumes that the shell buckles in the elastic domain.

(2) The upper boundary of the shell is assumed to be held circular, either by attachment to a roof or by a ring stiffener of appropriate size, as defined in D.5.3.4.2.

(3) A buckle that forms over only part of the wall height is termed an internal buckle, by contrast with a full height buckle that extends to the shell base.

(4) The distance from the top of the cylindrical wall to each change of plate thickness (and to the cylinder base) should be identified as h_i, where i represents the number of the lowest uniform thickness section within the chosen height h_i, beginning with i = 1 for the top section and finishing with $i = n$ for the bottom section (Figure D.5), so h_n is the total height of the shell wall.

(5) The distance h_i for a shell zone is thus given by

$$h_i = \sum_{j=1,i} l_j \qquad \text{(D.83)}$$

(6) The buckling assessment examines the possibility that each change of plate thickness can be the approximate location of the base of a potential buckle. Each potential buckle height should be assessed separately.

(7) For each buckle height h_m (m = 1, n), the critical external buckling pressure should be determined. The lowest calculated critical pressure then identifies both the critical pressure and the critical buckling mode.

NOTE Identification of the critical buckling mode is valuable in determining where an additional stiffening ring can be required.

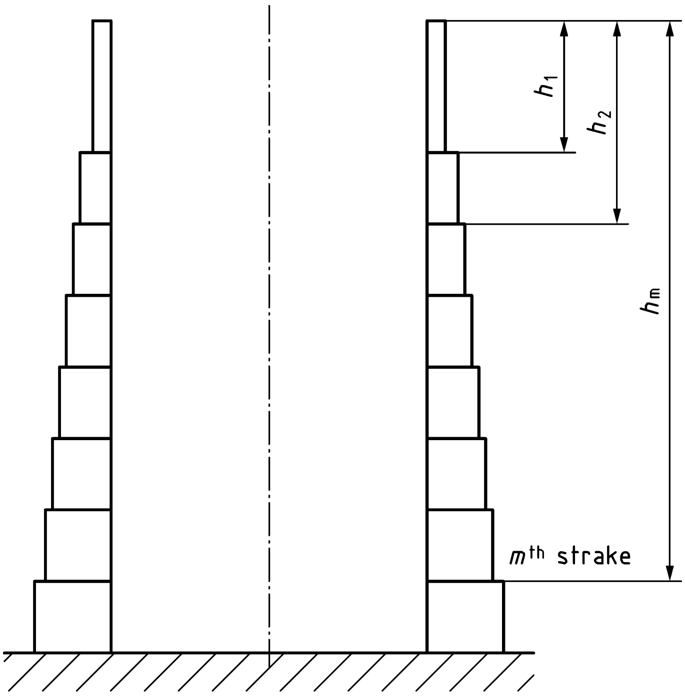


Figure D.5 — Stepped cylinder notation

(8) For each potential buckle extending to the bottom of the m^{th} section (Figure D.5), the following procedure should be followed.

(9) A buckle that extends from the top to the bottom of the m^{th} section, the heights from the top to the base of each strake should be identified as $h_1, h_2, ..., h_m$. The thicknesses of the lowest sections of these parts are denoted $t_1, t_2, ..., t_m$. The height of the potential buckle is given by h_m.

For each section within the buckle, the value of H_i is found as:

$$\mathrm{H}_i = \left(h_i - \frac{h_m}{2\pi} \sin \frac{2\pi h_i}{h_m} \right) \tag{D.84}$$

The equivalent thickness $t_{eq.m}$ of this potential buckle is found as:

$$t_{eq,m} = \left\{ \left(\frac{1}{h_m} \right) \sum_{i=1}^{m} \left[t_i^3 \left(\mathrm{H}_i - \mathrm{H}_{i-1} \right) \right] \right\}^{0,333} \tag{D.85}$$

with $H_0 = 0$ when $i = 1$.

(10) Because the base of an internal buckle is axially restrained by the shell below it, but is not axially restrained at the top, the effective value of C_θ for an internal buckle is C_θ = 1,25 (see Table D.4).

(11) Where the potential buckle extends to the base of the stepped wall cylinder, the value of h_m should be taken as the height L. The value of C_θ should then be taken as C_θ = 1,0.

(12) The length category of the potential buckle is determined as follows:

a) The potential buckle height is in the medium or long length cylinder categories if

$$\omega_m \geq 25 \qquad \text{where} \qquad \omega_m = \frac{h_m}{\sqrt{r t_{eq,m}}} \tag{D.86}$$

b) The potential buckle height is in the short cylinder category if:

$$\omega_m < 25 \tag{D.87}$$

(13) For a medium or long length internal potential buckle, the critical buckling pressure should be calculated as:

$$q_{Rcr,m} = 1,15 \frac{E}{\omega_m} \left(\frac{t_{eq,m}}{r} \right)^2 \tag{D.88}$$

which incorporates the factor C_θ = 1,25.

NOTE 1 Both Formula (D.88) and all the subsequent paragraphs in this sub-clause have been drafted in such a manner that it is not necessary for the designer to find an appropriate value for C_θ, as this boundary condition effect is already included within each formula.

NOTE 2 Both Formulae (D.88) and (D.89) are valid for buckles that extend to the base, provided that the base is anchored or supported as a BC1 boundary condition and that the top is radially restrained as a BC2 boundary condition.

(14) For a short length internal potential buckle, the critical buckling pressure should be calculated as:

$$q_{Rcr,m} = \left(1,15 + \frac{7,4}{\omega_m^2} - \frac{3,7}{\omega_m^3} \right) \frac{E}{\omega_m} \left(\frac{t_{eq,m}}{r} \right)^2 \tag{D.89}$$

(15) For a medium or long length cylinders where the base is not axially restrained (unanchored base) and with a potential full height buckle (extending to the base), the critical buckling pressure should be calculated as:

$$q_{Rcr,n} = 0,92 \frac{E}{\omega_n} \left(\frac{t_{eq,n}}{r} \right)^2 \tag{D.90}$$

where n represents the final shell segment.

NOTE The above treatment is very conservative for long cylinders, but the majority of applications where this is used are intended to involve shells of medium length.

(16) For a short length cylinder where the base is not axially restrained (unanchored base) and with a full height potential buckle (extending to the base), the critical buckling pressure should be calculated as:

$$q_{Rcr,n} = \left(1,00 + \frac{3}{\omega_n^{1,35}} \right) \frac{E}{\omega_n} \left(\frac{t_{eq,n}}{r} \right)^2 \tag{D.91}$$

(17) The critical buckling pressure for the complete cylindrical shell should be found as:

$$q_{Rcr} = \min(q_{Rcr,i}, i = 1, n) \tag{D.92}$$

NOTE The value of i that leads to the minimum value q_{Rcr} is often required for later use.

(18) The buckle height corresponding to $q_{Rcr} = \min(q_{Rcr,i}, i = 1, n)$ should be noted and identified as h_{cr}.

(19) Where a combination of loading makes it necessary to identify the critical circumferential stress associated with this critical pressure, the critical circumferential stress should be taken as different in each segment and defined as:

$$\sigma_{Rcr,m} = q_{Rcr,m} \left(\frac{r}{t_{eq,m}} \right) \tag{D.93}$$

(20) The characteristic value of the buckling pressure for the complete cylindrical shell should be found as:

$$q_{Rk} = \alpha_{\theta G} \alpha_{\theta I} q_{Rcr} / \gamma_{M1} \tag{D.94}$$

(21) The geometric reduction factor $\alpha_{\theta G}$ should be taken from Formulae (D.66) to (D.68) using the length h_{cr} found in (18).

(22) The imperfection reduction factor $\alpha_{\theta I}$ should be found using Formula (D.29), together with the appropriate Fabrication Quality Class as defined in Table D.6.

D.5.3.2 Critical circumferential buckling pressure under wind

(1) Following the procedure of D.5.3.1, the height of each potential buckle is h_{m}.

NOTE In this procedure, the reference condition for each buckle is a medium length cylinder with unanchored base, leading to the reference critical pressure $q_{\mathrm{Rcr,m}}$.

(2) The reference critical uniform pressure for this potential buckle should be calculated as:

$$q_{u,Rcr,m} = 0,92 \frac{E}{\omega_m} \left(\frac{t_{eq,m}}{r} \right)^2 \tag{D.95}$$

(3) For the wind buckling assessment the relative length of the potential buckle is defined as:

$$\Omega_m = \left(\frac{h_m}{r} \right) \left(\frac{t_{eq,m}}{r} \right)^{1/2} \tag{D.96}$$

(4) The critical buckling wind stagnation pressure for this potential buckle is given by:

$$q_{w,Rcr,m} = q_{u,Rcr,m} \left\{ 0,83 + 1,64 \Omega_m^{0,23} \right\} \quad \text{for} \quad \Omega_m < 0,40 \tag{D.97}$$

$$q_{w,Rcr,m} = q_{u,Rcr,m} \left\{ 0,55 + 0,705 \Omega_m^{-0,9} \right\} \quad \text{for} \quad 0,40 \le \Omega_m < 1,40 \tag{D.98}$$

$$q_{w,Rcr,m} = 1,07 q_{u,Rcr,m} \quad \text{for} \quad 1,40 \le \Omega_m \tag{D.99}$$

(5) The relative length required to define the effect of geometric nonlinearity is calculated from:

$$\xi_m = \left(\frac{h_m}{r} \right) \left(\frac{t_{eq,m}}{r} \right)^{4/7} \tag{D.100}$$

(6) The sensitivity to geometric nonlinearity α_G is given by:

$$\alpha_{G,m} = 1,0 \quad \text{for} \quad \xi_m \le 0,161 \tag{D.101}$$

$$\alpha_{G,m} = \left(\frac{0,1}{2,12 \xi_m^{0,06} - 1,8} \right) \quad \text{for} \quad 0,161 < \xi_m < 0,344 \tag{D.102}$$

$$\alpha_{G,m} = 0,53 \quad \text{for} \quad 0,334 \le \xi_m \tag{D.103}$$

(7) The imperfection reduction factor $\alpha_{\theta I}$ should be found using Formula (D.29), together with the appropriate Fabrication Quality Class as defined in Table D.6.

(8) The characteristic value of the buckling pressure of this potential buckle should be found as:

$$q_{w,Rk,m} = \alpha_{\theta I} \alpha_{G,m} \; q_{w,Rcr,m} \tag{D.104}$$

(9) Considering all the different potential buckle lengths h_m, the characteristic value of the buckling pressure for the complete cylindrical shell under wind should be found as:

$$q_{w,Rk} = \min(q_{w,Rk,i}, i = 1, n) \tag{D.105}$$

(10) The corresponding actual buckle height h_k should be found using the value of *i* in Formula (D.105) that leads to the minimum value of $q_{w,Rk,i}$.

NOTE The potential buckle heights h_i are needed to establish the required location for a secondary ring, if that is needed.

(11) The design value of the stagnation pressure of the wind at the eaves should be identified as $q_{w,Ed}$.

(12) The total inward pressure at the stagnation location should be evaluated as:

$$q_{net,Ed} = q_{w,Ed} + q_{s,Ed} \tag{D.106}$$

where

$q_{w,Ed}$ is the stagnation pressure at the eaves at the windward location (Figure D.2);

$q_{s,Ed}$ is the internal suction caused by venting, internal partial vacuum or other phenomena.

(13) Formula (D.25) and Tables D.4 and D.5 should not be used in conjunction with D.5.3.2.

NOTE This process in D.5.3.2 is valid for short and medium length cylindrical shells and conservative for long cylinders. If the higher resistance for short cylinders under uniform pressure (Formula (D.25)) is adopted into this process, the increased resistance of short cylinders is accounted for twice, making the result unsafe.

(14) Where this load case must be combined with other stress components, the circumferential design stress to be introduced into 9.5 should be determined as:

$$\sigma_{\theta,Ed} = q_{net,Ed}\left(\frac{r}{t}\right) \tag{D.107}$$

D.5.3.3 Buckling strength verification for circumferential compression in a stepped wall

(1) The following checks should be made, as appropriate:

$$q_{Ed} \leq q_{Rk} / \gamma_{M1} \tag{D.108}$$

or

$$q_{w,Ed} \leq q_{w,Rk} / \gamma_{M1} \tag{D.109}$$

or

$$q_{net,Ed} \leq q_{w,Rk} / \gamma_{M1} \tag{D.110}$$

where

q_{Ed} is the design value of uniform external pressure;

q_{Rk} is the characteristic value of the buckling resistance pressure for uniform external pressure;

$q_{net,Ed}$ is the design value of the total inward pressure at the stagnation location;

$q_{w,Rk}$ the characteristic value of the buckling resistance pressure under wind loading.

D.5.3.4 Stiffening rings to resist buckling under external pressure and wind

D.5.3.4.1 General

(1) A cylindrical shell with a fixed roof supported on a roof structure may be considered to be adequately stiffened at the top of the shell by the roof structure. A primary ring may be omitted.

(2) An open top cylindrical shell should be provided with a primary ring which is located at or near the top of the uppermost course that fulfils the requirements given below.

(3) If the lower edge of the cylindrical shell is effectively anchored to resist vertical displacements, the primary stiffening ring may be designed by satisfying both the strength and the stiffness requirements given in this sub-clause.

(4) If the lower edge of the shell is not effectively anchored to resist vertical displacements the buckling assessment should be carried out using computational analysis as defined in 9.8.

D.5.3.4.2 Stiffness requirement for the top ring

(1) The top ring must have an adequate elastic stiffness to provide an appropriate boundary condition for the buckle in the stepped wall shell. The minimum circumferential bending stiffness $EI_{r\theta}$ of the ring should be obtained from the following evaluations.

(2) The required second moment of area of the top ring should be evaluated as:

$$I_{r\theta,\min} = k_{\min} h_m t_{eq,m}^3 \tag{D.111}$$

where h_m and $t_\mathrm{eq,m}$ are found using D.5.3.1 and k_min is given by:

$$k_{\min} = k_0 + k_1 \left(\frac{h_m}{r} \right) + k_2 \left(\frac{h_m}{r} \right)^2 \tag{D.112}$$

in which:

$$k_0 = 23010 \left(\frac{t}{r} \right)^{1,53} \tag{D.113}$$

$$k_1 = -5,9 \times 10^5 \left(\frac{t}{r} \right)^2 \tag{D.114}$$

$$k_2 = 2 \times 10^6 \left(\frac{t}{r} \right)^{2,4} \tag{D.115}$$

NOTE Formulae (D.112) to (D.115) are valid for the range $400 \le r/t \le 2000$ and $0,5 \le L/r \le 5$.

(3) For L/r < 0,5, k_min should be assumed to rise very rapidly as L/r decreases.

(4) Alternative simpler measures may be used, as shown in Table D.8.

Table D.8 — Simple values of the minimum flexural stiffness of a top ring

$L/r < 0,5$	$500 \le r/t \le 1000$ with $0,5 \le L/r \le 2$	$r/t > 1000$ with $0,5 \le L/r \le 2$	All $r/t > 500$ with $L/r > 2$
k_min = 0,1	k_min = 1,0	k_min = 0,48	k_min = 20

D.5.3.4.3 Bending moments in the top ring

(1) The bending moments in the top ring may be assessed using computational analysis. Where this is not used, the following procedure should be used for cylindrical shells with aspect ratio (height-to-diameter ratio) less than or equal to 1,0. It may be used for higher aspect ratios, but has not yet been verified for them.

(2) The peak value of the horizontal (circumferential) bending moment about the vertical axis of the top ring, accounting for the interaction between the shell and the ring can be found as follows:

$$M_{\theta,Est}\left(\theta\right) = q_{w,Ed} h_m r_G^2 \quad \sum_{j=2}^{n} \frac{C_j f_2\left(\kappa_j\right)}{(j^2 - 1)} f_1\left(\kappa_j\right) \cos j\theta \tag{D.116}$$

where

$q_{w,Ed}$	is the design value of the wind inward pressure at the stagnation location;
r_G	is the radius of the ring centroid;
h_m	is the height of the potential buckle (see D.5.3.1);
$f_1(\kappa_j)$	is the shell-ring interaction function;
$f_2(\kappa_j)$	is the tributary height (h_{eff}) coefficient;
κ_j	is the shell-ring stiffness ratio for harmonic *j;*
C_j	is the harmonic coefficient for harmonic *j* of wind loading;
j	is the harmonic number of the component of wind pressure ($j \geq 2$);
θ	is the circumferential angle measured from the windward direction ($\theta = 0$ on the stagnation meridian).

(3) The shell-ring interaction $f_1(\kappa_j)$ for harmonic *j* is given by:

$$f_1\left(\kappa_j\right) = \frac{1}{\left(1+\kappa_j\right)^{0,95}} \tag{D.117}$$

(4) The tributary height coefficient $f_2(\kappa_j)$ for harmonic *j* is given by:

$$f_2\left(\kappa_j\right) = 0,5\left(\frac{22-j}{\sqrt{\kappa_j}+200}\right)\left(10,7-\frac{h_m}{r}\right) \tag{D.118}$$

(5) The shell to ring stiffness ratio (κ_j) for harmonic *j* is given by:

$$\kappa_j = \left(\frac{1}{j^2\left(j^2-1\right)^2}\right)\left(\frac{3t_{eq,m} r_G^6}{h_m I_{r\theta}\left[6,9 r_G^2 + j^2 h_m^2\right]}\right) \tag{D.119}$$

where

$I_{r\theta}$	is the second moment of area of the ring for circumferential bending (bending about the vertical axis);
$t_{eq,m}$	is the equivalent thickness of the shell wall in the critical buckling mode (see D.5.3.1).

(6) The harmonic coefficients for the shell should be taken from EN 1991-1-4 for wind loads on a shell of the appropriate full aspect ratio (L/r).

NOTE 1 Where a quick assessment is required, the constants for $n = 4$ can give a suitable result: $C_2 = +1,0$, $C_3 = +0,45$ and $C_4 = -0,15$. ($C_1 = 0$ always).

NOTE 2 The peak circumferential bending moment does not always occur at the stagnation point (θ = 0). A full circumferential evaluation can sometimes be needed.

D.5.4 Shear

D.5.4.1 Critical shear buckling stresses

(1) If no specific rule for evaluating an equivalent single cylinder of uniform wall thickness is available, the formulae of D.3.5.2 (1) to (5) may be applied.

(2) The further determination of the elastic critical shear buckling stresses may, in principle, be performed as in D.5.3.1, but replacing the external pressure formulae, transformed into circumferential compressive stresses, by the relevant shear formulae from D.3.5. Where this is performed, it is recommended that an LBA analysis is used instead to determine the critical shear stress more accurately for the specific loading condition (see 9.7).

D.5.4.2 Buckling strength verification for shear

(1) The rules of D.5.3.1 to D.5.3.3 may be applied, but replacing the external pressure, transformed into a circumferential stress, and applying the formulae with the relevant shear resistance formulae (see D.3.5.2).

D.6 Lap jointed cylindrical shells

D.6.1 General

D.6.1.1 Definitions

D.6.1.1.1 Circumferential lap joint

A lap joint that runs in the circumferential direction around the shell axis.

D.6.1.1.2 Meridional lap joint

A lap joint that runs parallel to the shell axis (meridional direction).

D.6.1.2 Geometry and stress resultants

(1) Where a cylindrical shell is constructed using lap joints (see Figure D.6), the following provisions may be used in place of those set out in D.3.

(2) The following provisions apply both to lap joints that increase, and to those that decrease the radius of the middle surface of the shell.

(3) Where the lap joint runs in a circumferential direction around the shell axis (circumferential lap joint), the provisions of D.6.2 should be used for axial compression.

(4) Where many lap joints at different axial coordinates run in a circumferential direction around the shell axis (circumferential lap joints) with changes of plate thickness down the shell, the provisions of D.5.3 should be used for circumferential compression.

(5) Where a continuous lap joint runs parallel to the shell axis (unstaggered meridional lap joint), the provisions of D.5.3 should be used for circumferential compression.

(6) In other cases, no special considerations are needed for the influence of lap joints on the buckling resistance.

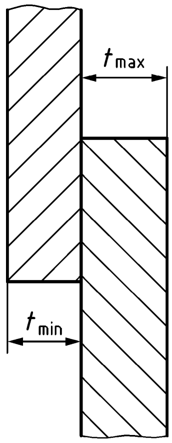


Figure D.6 — Lap jointed shell

D.6.2 Axial (meridional) compression

(1) Where a lap jointed cylinder is subject to axial compression, with circumferential lap joints, the buckling resistance may be evaluated as for a uniform or stepped-wall cylinder, as appropriate, but with the value of α_x (Formula (D.11)) should be reduced by the multiplying factor k_L to:

$$\alpha_{xL} = k_L \alpha_x \tag{D.120}$$

$$k_L = 1 - 0,595 \left\{ \frac{1 - 0,61\sqrt{\delta_0 / t}}{1 + 4(\delta_0 / t)^2} \right\} \tag{D.121}$$

The value of δ_0/t should be calculated using Formula (D.14).

(2) Where a change of plate thickness occurs at the lap joint, the design buckling resistance should be taken as the value for the thinner plate as determined in (1).

D.6.3 Circumferential (hoop) compression

(1) Where a lap jointed cylinder is subject to circumferential compression across continuous meridional lap joints, the design buckling resistance may be evaluated as for a uniform or stepped-wall cylinder, as appropriate, but with a reduction factor of 0,90.

(2) Where a lap jointed cylinder is subject to circumferential compression, with many circumferential lap joints and a changing plate thickness down the shell, the procedure of D.5.3 should be used without the geometric restrictions on joint eccentricity, and with the design buckling resistance reduced by the factor 0,90.

(3) Where lap joints are used in both directions, with staggered placement of the meridional lap joints in alternate strakes or courses (as in brickwork construction), the design buckling resistance should be evaluated as in (2), but no further resistance reduction need be applied.

D.6.4 Shear

(1) Where a lap jointed cylinder is subject to membrane shear, the buckling resistance may be evaluated as for a uniform or stepped-wall cylinder, as appropriate, without any special allowance for the lap joints.

D.7 Complete and truncated conical shells

D.7.1 General

D.7.1.1 Notation

(1) In this Clause the following notation is used:

h is the axial length (height) of the truncated cone;

L is the meridional length of the truncated cone ($=h/\cos\beta$);

r is the radius of the cone middle surface, perpendicular to axis of rotation, that varies linearly down the length;

r_1 is the radius at the small end of the cone;

r_2 is the radius at the large end of the cone;

β is the apex half angle of cone.

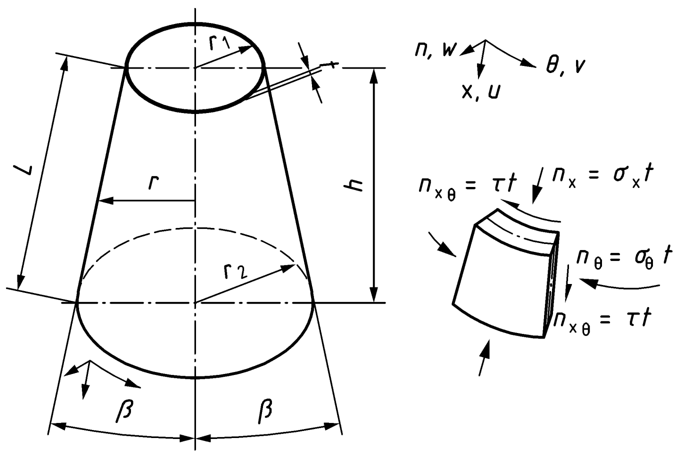


Figure D.7 — Cone geometry, membrane stresses and stress resultants

D.7.1.2 Boundary conditions

(1) The following formulae should be used only for shells with boundary conditions BC1 or BC2 at both edges (see 6.2.2.2 and 9.3), with no distinction made between them. They should not be used for a shell in which any boundary condition is BC3.

(2) The rules in this D.7 should be used only for the two following radial and normal displacement boundary conditions, at either end of the cone:

"cylinder condition" $w = 0$ radially restrained (D.122)

"ring condition" $u \sin\beta + w \cos\beta = 0$ normal displacements at boundary restrained (D.123)

D.7.1.3 Geometry

(1) Only truncated cones of uniform wall thickness and with apex half angle $\beta \leq 65°$ (see Figure D.7) are covered by the following rules.

D.7.2 Design buckling stresses

D.7.2.1 Equivalent cylinder

(1) The design buckling stresses that are needed for the buckling strength verification according to 9.5 may all be found by treating the conical shell as an equivalent cylinder of length ℓ_e and of radius r_e in which ℓ_e and r_e depend on the type of membrane stress distribution in the conical shell.

D.7.2.2 Meridional compression

(1) For cones under meridional compression, the equivalent cylinder length ℓ_e should be taken as:

$$\ell_e = L \tag{D.124}$$

(2) The equivalent cylinder radius at any buckling relevant location r_e should be taken as:

$$r_e = \frac{r}{\cos\beta} \tag{D.125}$$

(3) The characteristic imperfection amplitude δ_k, which can be needed for tolerance controls, should be taken as:

$$\frac{\delta_o}{t} = \left(\frac{22}{Q}\right)\overline{\lambda} \tag{D.126}$$

in which Q is the meridional compression fabrication quality parameter, t is the local thickness and $\overline{\lambda}$ is the shell relative slenderness. The values of $\overline{\lambda}$ and Q should be taken as those for the equivalent cylinder, with the value of Q taken from Table D.1.

NOTE Meridional compression strictly means the compressive membrane stress in the sloping meridional direction, which is not the same as an applied axial force divided by the cross-sectional area of the shell (see Annex A.5).

D.7.2.3 Circumferential (hoop) compression

(1) For cones under circumferential compression, the equivalent cylinder length ℓ_e should be taken as:

$$\ell_e = L \tag{D.127}$$

(2) The equivalent cylinder radius r_e should be taken as:

$$r_e = \frac{(r_1 + r_2)}{2\cos\beta} \tag{D.128}$$

D.7.2.4 Uniform external pressure

(1) For cones under uniform external pressure q, that have either the boundary conditions BC1 at both ends or the boundary conditions BC2 at both ends, the following procedure may be used to produce a more economic design.

(2) The equivalent cylinder length ℓ_e should be taken as the lesser of:

$$\ell_e = L \tag{D.129}$$

and

$$\ell_e = \left(\frac{r_2}{\sin\beta}\right)(0{,}53 + 0{,}125\beta) \qquad \text{(D.130)}$$

where the cone apex half angle β is measured in radians.

(3) For shorter cones, where the equivalent length ℓ_e is given by Formula (D.129), the equivalent cylinder radius r_e should be taken as:

$$r_e = \left(\frac{0{,}55 r_1 + 0{,}45 r_2}{\cos\beta}\right) \qquad \text{(D.131)}$$

(4) For longer cones, where the equivalent length ℓ_e is given by Formula (D.130), the equivalent cylinder radius r_e should be taken as:

$$r_e = 0{,}71\, r_2 \left[\frac{1 - 0{,}1\beta}{\cos\beta}\right] \qquad \text{(D.132)}$$

where β is measured in radians.

(5) The buckling strength verification (see 9.5) should be based on the notional circumferential membrane stress:

$$\sigma_{\theta,Ed} = q\left(\frac{r_e}{t}\right) \qquad \text{(D.133)}$$

in which q is the external pressure, and no account is taken of the meridional membrane stress induced by the external pressure.

D.7.2.5 Shear

(1) For cones under membrane shear stress, the equivalent cylinder length ℓ_e should be taken as:

$$\ell_e = h \qquad \text{(D.134)}$$

(2) The equivalent cylinder radius r_e should be taken as:

$$r_e = \left[1 + \rho_g - \frac{1}{\rho_g}\right] r_1 \cdot \cos\beta \qquad \text{(D.135)}$$

in which:

$$\rho_g = \sqrt{\frac{r_1 + r_2}{2 r_1}} \qquad \text{(D.136)}$$

(3) The buckling strength verification (see 9.5) should be based on the maximum membrane shear stress in the shell.

D.7.2.6 Uniform torsion

(1) For cones under membrane shear stress, where this is produced by uniform torque (inducing a shear that varies quadratically down the meridian: see Table A.6), the following procedure may be used to produce a more economic design, provided the boundary conditions are BC2 at both ends and the parameter ρ_u satisfies the condition $\rho_u \leq 0{,}8$.

$$\rho_u = \frac{L \sin \beta}{r_2} \tag{D.137}$$

(2) The equivalent cylinder length ℓ_e should be taken as:

$$\ell_e = L \tag{D.138}$$

(3) The equivalent cylinder radius r_e should be taken as:

$$r_e = \left(\frac{r_1 + r_2}{2 \cos \beta} \right) \left(1 - \rho_u^{2,5} \right)^{0,4} \tag{D.139}$$

(4) The buckling strength verification (see 9.5) should be based on the maximum membrane shear stress in the shell.

D.7.3 Buckling strength verification

D.7.3.1 Meridional compression

(1) The buckling design check should be carried out at that point of the cone where the combination of design meridional membrane stress $\sigma_{x,Ed}$ and design meridional buckling stress $\sigma_{x,Rd}$ according to D.7.2.2 is most critical.

(2) In the case of meridional compression caused by a constant axial force on a truncated cone, both the small radius r_1 and the large radius r_2 should be considered as possible locations for the most critical position.

(3) In the case of meridional compression caused by a constant global bending moment on the cone, the small radius r_1 should be taken as the most critical.

(4) The design meridional buckling stress $\sigma_{x,Rd}$ should be determined for the equivalent cylinder according to D.3.3.

D.7.3.2 Circumferential (hoop) compression and uniform external pressure

(1) Where the circumferential compression is caused by uniform external pressure, the buckling design check should be carried out using the design circumferential membrane stress $\sigma_{\theta,Ed}$ and the design circumferential buckling stress $\sigma_{\theta,Rd}$ according to D.7.2.1 and D.7.2.3 or D.7.2.4.

(2) Where the circumferential compression is caused by actions other than uniform external pressure, the calculated stress distribution $\sigma_{\theta,Ed}(r)$ should be replaced by a fictitious enveloping stress distribution $\sigma_{\theta,Ed,env}(r)$ that everywhere exceeds the calculated value, but which would arise from a fictitious uniform external pressure. The buckling design check should then be carried out as in paragraph (1), but using $\sigma_{\theta,Ed,env}$ as it varies with $r_e\cos\beta$, instead of $\sigma_{\theta,Ed}$.

(3) The design buckling stress $\sigma_{\theta,Rd}$ should be determined for the equivalent cylinder according to D.3.4.

D.7.3.3 Shear and uniform torsion

(1) In the case of shear caused by a constant global torque on the cone, the buckling design check should be carried out using the design membrane shear stress $\tau_{x\theta,Ed}$ at the point with $r = r_e \cos\beta$ and the design buckling shear stress $\tau_{x\theta,Rd}$ according to D.7.2.1 and D.7.2.5 or D.7.2.6.

(2) Where the shear is caused by actions other than a constant global torque (such as a global shear force on the cone), the calculated stress distribution $\tau_{x\theta,Ed}(r)$ should be replaced by a fictitious enveloping stress distribution $\tau_{x\theta,Ed,env}(r)$ that everywhere exceeds the calculated value, but which would arise from a fictitious global torque. The buckling design check should then be carried out as in (1), but using $\tau_{x\theta,Ed,env}$ as it varies with $r_e\cos\beta$, instead of $\tau_{x\theta,Ed}$.

(3) The design shear buckling stress $\tau_{x\theta,Rd}$ should be determined for the equivalent cylinder according to D.3.5.

Annex E
(normative)

Formulae to determine the buckling resistance of unstiffened shells when using reference resistance design

E.1 Use of this annex

(1) This Normative Annex contains additional formulae to determine the buckling resistance of unstiffened shells when using reference resistance design.

E.2 Scope and field of application

(1) This Normative Annex gives rules that apply to uniform cylindrical shells subjected to global bending, including combinations of bending with axial load.

NOTE The stress value of the buckling resistance of a cylindrical shell under global bending has long been assumed to be similar to that for uniform axial compression. Whilst this is substantially true when buckling occurs under elastic conditions, minor yielding has a major impact on the buckling resistance under global bending, making attainment of the full plastic moment more difficult than has sometimes been previously thought. The explanation for this phenomenon is given in Bibliography [7]. Cylindrical shells under global bending in the elastic-plastic domain can have a lower apparent resistance than is implied by the use of Annex D rules for axial compression.

E.3 Cylindrical shells under global bending

E.3.1 General

E.3.1.1 Notation

(1) In this sub-clause the following notation is used (see Figure E.1):

r is the radius of the cylinder middle surface;

t is the uniform thickness of the cylinder;

L is the length of the cylinder;

M is the bending moment acting on the cylinder.

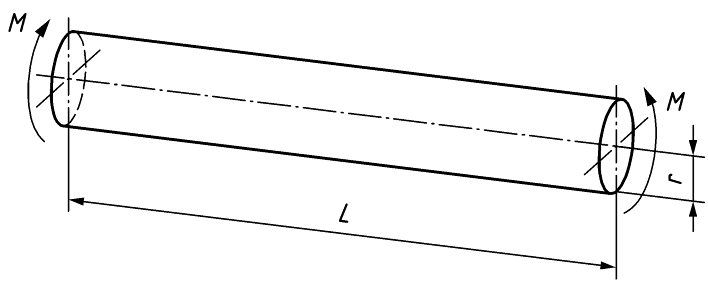


Figure E.1 — Cylinder under uniform global bending

E.3.1.2 Boundary conditions

(1) The rules given here apply to cylinders with fixed end boundary conditions BC1r and BC1f.

E.3.1.3 Loading conditions

(1) The following rules apply to global bending, characterised by the maximum moment M (see Figure E.1).

E.3.1.4 Length characterisation

(1) The first relative length is defined as:

$$\omega = \frac{L}{\sqrt{rt}} \tag{E.1}$$

(2) The second relative length of the cylinder Ω is defined as:

$$\Omega = \frac{L}{r}\sqrt{\frac{t}{r}} = \omega\frac{t}{r} \tag{E.2}$$

E.3.2 Buckling resistance under uniform global bending

E.3.2.1 Reference plastic resistance

(1) The reference plastic moment should be obtained from:

$$M_{\mathrm{R,pl}} = 4r^2 t\, f_{\mathrm{y,k}} \tag{E.3}$$

E.3.2.2 Reference elastic critical buckling resistance

(1) The reference elastic critical buckling moment $M_{\mathrm{R,cr}}$ is given by:

$$M_{\mathrm{R,cr}} = 1{,}90Ert^2 \tag{E.4}$$

NOTE The precise value of $M_{R.cr}$ is affected a little by the end boundary conditions, rising slightly at shorter lengths.

E.3.2.3 Length domains

(1) Under uniform global bending, a cylinder is classed as of medium length if:

$$\Omega < 0{,}5 \tag{E.5}$$

(2) Under uniform global bending, a cylinder is classed as long if:

$$\Omega \geq 0{,}5 \tag{E.6}$$

NOTE In formal terms, cylinders of lengths between $\Omega = 0{,}5$ and $\Omega = 7{,}0$ are classed as of “transitional length” because the effect of ovalisation begins at the lower limit and occurs fully above the higher limit. However, the formulae given here cover both this domain and formally long cylinders, so the term “transitional” is not used in this standard.

(3) Cylinders need not be checked against shell buckling under pure bending if they satisfy:

$$\bar{\lambda}_b \leq \bar{\lambda}_{b0} \tag{E.7}$$

where $\overline{\lambda}_b$ is given by Formula (E.11) and $\overline{\lambda}_{b0}$ is given by Formula (E.18).

E.3.2.4 Buckling capacity parameters

(1) The reduced reference plastic moment, accounting for imperfections, is given by:

$$M_{R,pl,I} = \left[0,20 + \frac{0,80}{1 + 0,23\left(\delta_0 / t\right)^2} \right] M_{R,pl} \tag{E.8}$$

in which δ_0 is the imperfection amplitude:

$$\frac{\delta_0}{t} = \frac{1}{Q_b}\sqrt{\frac{r}{t}} \tag{E.9}$$

where

Q_b is the fabrication quality parameter for global bending given in Table E.1.

Table E.1 — Values of fabrication quality parameter Q_b

Quality Class	Description	Q_b
Class A	excellent	40
Class B	high	25
Class C	normal	16

(2) The reference plastic resistance $F_{R,pl}$ may be taken as the value $M_{R,pl,I}$ (Formula (E.8)) to allow for the effect of imperfections. The reference elastic critical buckling resistance $F_{R,cr}$ may be taken as $M_{R,cr}$ (Formula (E.4)).

(3) The reference resistances of the cylinder are then given by:

$$R_{pl} = \frac{M_{R,pl,I}}{M_{Ed}} \quad \text{and} \quad R_{cr} = \frac{M_{R,cr}}{M_{Ed}} \tag{E.10}$$

(4) The relative slenderness $\overline{\lambda}_b$ is given by:

$$\overline{\lambda}_b = \sqrt{\frac{R_{pl}}{R_{cr}}} = \sqrt{\frac{M_{R,pl,I}}{M_{R,cr}}} \tag{E.11}$$

(5) The geometrical reduction factor α_{bG} should be determined as:

$$\alpha_{bG} = 0,9 \quad \text{when} \quad \Omega < 0,5 \tag{E.12}$$

$$\alpha_{bG} = 0,5 + (0,38\sin\psi + 0,48\cos\psi) * e^{-0,94\psi} \quad \text{when} \quad \Omega \geq 0,5 \tag{E.13}$$

where

$$\psi = 0,85\ \Omega$$

(6) The imperfection reduction factor α_{bI} should be found as:

$$\alpha_{bI} = \frac{1}{1 + \left(0,70 + \dfrac{1,05}{1 + 0,42\Omega^{2,8}}\right)\left(\delta_0 / t\right)^{0,7}} \tag{E.14}$$

(7) The elastic buckling reduction factor α_b is defined as:

$$\alpha_b = \alpha_{bI}\alpha_{bG} \tag{E.15}$$

(8) The plastic range factor β_b should be found as:

$$\beta_b = 1 - \left(\frac{0,785}{1 + 1,3\sqrt{\delta_0 / t}}\right) f_\Omega \tag{E.16}$$

in which

$$f_\Omega = 0,70 + \frac{0,44}{1 + 1,66\Omega^{1,87}} \quad \text{but} \quad \leq 1,0 \tag{E.17}$$

(9) The squash limit relative slenderness $\bar{\lambda}_{b0}$ should be found as:

$$\bar{\lambda}_{b0} = \left(\frac{0,3}{1 + 0,4\sqrt{\delta_0 / t}}\right) f_\Omega \tag{E.18}$$

(10) The interaction exponent η_b should be obtained from the two values η_{b0} and η_{bp}:

$$\eta_{b0} = 1,0 \quad \text{when} \quad \Omega < 4,5 \tag{E.19}$$

$$\eta_{b0} = 0,133\left(12 - \Omega\right) \quad \text{when} \quad 4,5 \leq \Omega < 7,5 \tag{E.20}$$

$$\eta_{b0} = 0,6 \quad \text{when} \quad \Omega \geq 7,5 \tag{E.21}$$

and:

$$\eta_{bp} = 0,08\left(7 - \Omega\right) \quad \text{when} \quad \Omega < 5 \tag{E.22}$$

$$\eta_{bp} = 0,16\left(\Omega - 4\right) \quad \text{when} \quad \Omega \geq 5 \tag{E.23}$$

combined using:

$$\eta_b = \left[\frac{\bar{\lambda}_b\left(\eta_{bp} - \eta_{bo}\right) + \bar{\lambda}_{bp}\eta_{bo} - \bar{\lambda}_{bo}\eta_{bp}}{\bar{\lambda}_{bp} - \bar{\lambda}_{bo}}\right] \tag{E.24}$$

(11) The hardening limit χ_{bh} should be taken as:

$$\chi_{bh} = 1,05 \tag{E.25}$$

E.3.2.5 Characteristic buckling resistance

(1) The characteristic buckling resistance should be determined according to 9.6.3 (Formulae (9.31) to (9.37)) with the leading load F_{Ed} taken as the applied bending moment M_{Ed}.

(2) The characteristic buckling resistance or the buckling moment is given by:

$$R_k = \chi R_{pl} \quad \text{or} \quad M_{R,k} = \chi M_{R,pl,I} \tag{E.26}$$

where:

χ is the elastic-plastic buckling reduction factor according to 9.6.3 (8).

(3) The buckling verification is then:

$$R_d = \frac{R_k}{\gamma_{M1}} \geq 1 \tag{E.27}$$

where the safety factor γ_{M1} should be as defined in 4.4.

E.3.3 Buckling resistance under global bending with axial compression

E.3.3.1 General

(1) The following rules apply only to cylinders of medium length, which are classed as those for which:

$$\Omega \leq 0{,}5 \tag{E.28}$$

NOTE: This restriction is intended to prevent the cylinder from being susceptible to cross-sectional ovalisation, whose manifestation under an interaction of global bending with axial compression is not yet fully understood. It is good practice to employ stiffening rings to satisfy this length restriction.

E.3.3.2 Interaction verification

(1) The design value of the buckling resistance under axial force is given by:

$$N_{R,d} = \frac{A\sigma_{x,Rk}}{\gamma_{M1}} \tag{E.29}$$

where

$\sigma_{x,Rk}$ is the characteristic buckling stress under uniform axial compression according to 9.5.2 (7);

γ_{M1} is the partial factor of shell stability as defined in 4.4.

(2) The design value of the buckling resistance under a bending moment is given by:

$$M_{R,d} = \frac{M_{R,k}}{\gamma_{M1}} \tag{E.30}$$

where

$M_{R,k}$ is the characteristic buckling moment according to Formula (E.26);

γ_{M1} is the partial factor of shell stability should be taken from 4.4.

(3) The buckling verification under combined global bending and axial compression should be made so as to satisfy:

$$\left(\frac{N_{Ed}}{N_{R,d}}\right)^{k_1} + \left(\frac{M_{Ed}}{M_{R,d}}\right)^{k_2} \leq 1 \tag{E.31}$$

in which N_{Ed} is the applied axial force ($N_{\text{Ed}} = A\sigma_{\text{x,Ed}}$) and M_{Ed} is the acting bending moment, and k_1 and k_2 are interaction parameters which should be both taken as $k_1 = 1$ and $k_2 = 1$.

NOTE: The restriction of the above interaction parameters to unity is conservative and is used to permit the design to benefit from the well-known favourable moment-axial force interaction relationship for a thick circular hollow section. This is achieved by making a cylindrical shell thicker than the formal scope limits.

E.4 Spherical dome shells

E.4.1 General

E.4.1.1 Scope

(1) The following rules apply to spherical dome shells under internal vacuum or uniform external pressure with different boundary conditions. The wall thickness of the spherical dome should not vary significantly. The shell is unstiffened.

(2) The rules are limited to the ranges given by:

$$100 \leq \frac{r_s}{t} \leq 2\,000 \tag{E.32}$$

$$15^\circ \leq \phi_0 \leq 45^\circ \tag{E.33}$$

(3) Spherical domes with geometries outside this range should be treated by computational GMNIA analysis.

(4) The shell segments should be connected by welded butt-joints or by bolted symmetrical double-lap-joints or the shell should consist of a single spherical element without any interior joints.

E.4.1.2 Notation

(1) In this sub-clause the following notation is used (see Figure E.2):

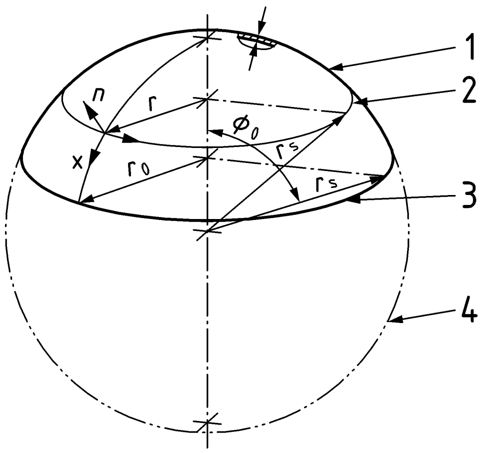
r_s is the radius of the sphere (shell middle surface);

r is the simple radius of the shell middle surface at any point = $r(x)$, perpendicular to the axis of rotation;

r_o is the simple radius of the base circle of a spherical dome;

t is the thickness of the shell;

ϕ_o is the semi-angle of a spherical dome (inclination at the outer support).



Key

1 spherical dome

2 circumference

3 base circle

4 complete sphere (outside the scope)

Figure E.2 — Spherical shell geometry

E.4.1.3 Support and boundary conditions

(1) The rules given here are applicable only to shells that are supported as indicated in Figure E.3 with the following boundary conditions:

SCr: spherical dome with clamped edges;

SCf*:* spherical dome with edges with displacement restraint in both the meridional direction and normal to the shell middle surface, and flexurally pinned.

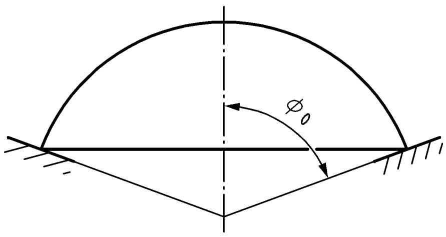
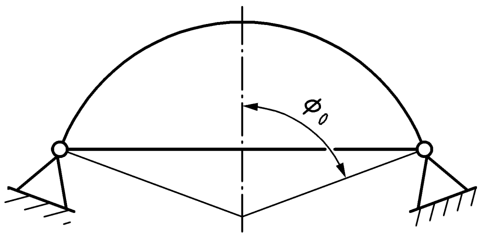
	
a) SCr	b) SCf

Figure E.3 — Illustrations of different support conditions

E.4.1.4 Loading conditions

(1) The following apply only to uniform internal vacuum or external pressure loading q perpendicular to the shell wall (see Figure E.4).

The design value of pressure q_{Ed} should be taken as the difference between the pressures on the inside and outside surfaces (both positive in the inwards).

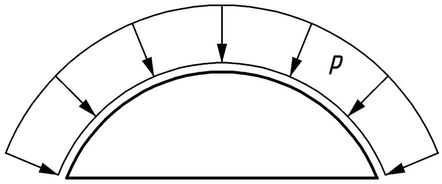


Figure E.4 — Spherical dome subjected to uniform external pressure

(2) For the loading cases of self-weight or snow, the procedures here may be used to obtain a conservative estimate of resistance if the value of the external pressure load q is taken as the maximum surface load normal to the middle surface of the shell.

E.4.2 Tolerances for spherical shells

(1) The geometrical tolerances are classified into three Fabrication Tolerance Quality Classes A to C.

(2) For the buckling relevant tolerances, the provisions of 9.4 apply by taking the radius r_s of the spherical shell in place of the cylinder radius r and the diameter $2r_s$ instead of the diameter d of the cylinder. The measurement of dimples (9.4.5) should be performed in both the meridional and circumferential directions using the gauge lengths ℓ_{gx} given by Formula (9.9) and ℓ_{gw} given by Formula (9.10), each of which must be curved to conform to the correct curvature of the sphere. It is not necessary to use the gauge length $\ell_{g\theta}$ given by Formula (9.11).

(3) The tolerance limits for each Fabrication Tolerance Quality Class given in 9.4 should be used.

E.4.3 Buckling design for uniform external pressure

E.4.3.1 Limitation on buckling calculations

(1) It is not necessary to check the resistance to buckling in spherical shells that satisfy the condition:

$$\frac{r_s}{t} \leq 0{,}605\left(\lambda_0^2\right)\frac{E}{f_{y,k}} \cdot \frac{C_{cr}}{C_{pl}} \tag{E.34}$$

where the squash limit relative slenderness λ_0 is defined in E.4.3.4 (6).

E.4.3.2 Reference elastic critical buckling resistance

(1) The reference elastic critical buckling pressure $q_{R,cr}$ is given by:

$$q_{R,cr} = \frac{2}{\sqrt{3\left(1-\nu^2\right)}} C_{cr\,cr} E\left(\frac{t}{r_s}\right)^2 \tag{E.35}$$

where the factor C_{cr} should be taken as C_{cr} =1 for both support conditions.

E.4.3.3 Reference plastic resistance

(1) The reference plastic resistance should be obtained from:

$$q_{R,pl} = 2 f_y C_{pl}\left(\frac{t}{r_s}\right) \tag{E.36}$$

in which the factor C_{pl} should be taken as C_{pl} =1 for both support conditions.

E.4.3.4 Buckling capacity parameters for simple conditions

(1) The relative slenderness $\overline{\lambda}$ is given by:

$$\overline{\lambda} = \sqrt{\frac{q_{R,pl}}{q_{R,cr}}} = \sqrt{\frac{R_{pl}}{R_{cr}}} \quad \text{(E.37)}$$

(2) The geometric reduction factor α_G should be taken as:

$$\text{SCr: } \alpha_{sG} = \frac{1}{1+1{,}33\varpi^{-0{,}68}} \quad \text{(E.38)}$$

$$\text{SCf: } \alpha_{sG} = \frac{1}{1+1{,}32\varpi^{-0{,}58}} \quad \text{(E.39)}$$

where

$$\varpi = \frac{r}{t}\left(1-\cos^{0{,}75}\phi_0\right) \quad \text{(E.40)}$$

(3) The imperfection reduction factor α_{sI} should be obtained from:

$$\alpha_{sI} = \frac{1}{1+2{,}36\left(\delta_0/t\right)^{0{,}86}} \quad \text{(E.41)}$$

in which δ_0 is the imperfection amplitude defined by:

$$\frac{\delta_0}{t} = \frac{1}{Q_s}\sqrt{\frac{r_s}{t}} \quad \text{(E.42)}$$

where

Q_S is the fabrication quality parameter for a spherical dome.

(4) The fabrication quality parameter Q_S should be taken from Table E.2 for the specified fabrication tolerance quality.

Table E.2 — Values of dome fabrication quality parameter Q_S

Quality class	Description	Q_S
Class A	excellent	40
Class B	high	25
Class C	normal	16

(5) The elastic buckling reduction factor α should be found as:

$$\alpha_s = \alpha_{sG}\alpha_{sI} \quad \text{(E.43)}$$

(6) The squash limit relative slenderness λ_0, the plastic range factor β, the interaction exponent η and the hardening limit χ_h should be taken as:

$$\text{SCr:} \quad \lambda_0 = 0{,}2; \quad \beta = 0{,}3; \quad \eta_0 = 1{,}5; \quad \eta_p = 1{,}2; \quad \chi_h = 1{,}01 \tag{E.44}$$

$$\text{SCf:} \quad \lambda_0 = 0{,}1; \quad \beta = 0{,}5; \quad \eta_0 = 1{,}5; \quad \eta_p = 1{,}0; \quad \chi_h = 1{,}01 \tag{E.45}$$

E.4.3.5 Characteristic buckling resistance

(1) The characteristic buckling resistance should be determined according to 9.6.3, with the leading load F_{Ed} taken as the applied external pressure q_{Ed}, the reference plastic resistance $F_{\mathrm{R,pl}}$ taken as $q_{\mathrm{R,pl}}$ (Formula (E.36)) and the reference elastic critical buckling resistance $F_{\mathrm{R,cr}}$ taken as $q_{\mathrm{R,cr}}$ (Formula (E.35)).

(2) The reference resistances are given by:

$$R_{\mathrm{pl}} = \frac{q_{\mathrm{R,pl}}}{q_{\mathrm{Ed}}} \quad \text{and} \quad R_{\mathrm{cr}} = \frac{q_{\mathrm{R,cr}}}{q_{\mathrm{Ed}}} \tag{E.46}$$

(3) The characteristic buckling resistance and the buckling pressure are given by:

$$R_{\mathrm{k}} = \chi R_{\mathrm{pl}} \quad \text{or} \quad q_{\mathrm{R,k}} = \chi q_{\mathrm{R,pl}} \tag{E.47}$$

where

χ is the elastic-plastic buckling reduction factor according to 9.6.3 (8).

E.4.4 Buckling strength verification for uniform external pressure

(1) The buckling verification is then:

$$R_{\mathrm{d}} = \frac{R_{\mathrm{k}}}{\gamma_{\mathrm{M1}}} \geq 1 \quad \text{or} \quad p_{\mathrm{R,d}} = \frac{p_{\mathrm{R,k}}}{\gamma_{\mathrm{M1}}} \geq p_{\mathrm{E,d}} \tag{E.48}$$

where the safety factor γ_{M1} is as defined in 4.4.

Bibliography

References contained in recommendations (i.e. through "should" clauses)

The following documents are referred to in the text in such a way that some or all of their content, although not requirements strictly to be followed, constitutes highly recommended choices or course of action of this document. Subject to national regulation and/or any relevant contractual provisions, alternative standards could be used/adopted where technically justified. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

None

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The following documents are referred to in the text in such a way that some or all of their content, although not requirements strictly to be followed, expresses a course of action permissible within the limits of the Eurocodes. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

None

Other references

The following documents are those not included in the above categories but are cited informatively in the document, for example in notes.

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